MEASURING k WITH A MODERN-DAY COULOMB EXPERIMENT

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1 INTRODUCTION

Figure 1. Portrait of Charles-Augustin De Coulomb holding his apparatus.

Charles-Augustin de Coulomb (1736–1806) was a French physicist who used a torsion balance to measure the attraction and repulsion between electric charges. In 1785, he published the inverse-square relationship now known as Coulomb's law. Today, his law remains a fundamental rule of electrostatics and provides the basis for calculating electric forces and fields [1]. Griffiths notes that Coulomb's law and superposition form the physical foundation of electrostatics and that nearly every force encountered in everyday life, apart from gravity, is electromagnetic in origin [2].

In this project, I attempted to reproduce Coulombs results using modern apparatus to gain a deeper understanding of his work and continue to recognize his genius roughly 240 years later.

2 THEORY

Coulomb's law states

$$F = k\frac{Q_1 Q_2}{R^2}. \qquad (1)$$

For the torsion balance, the electrostatic force is proportional to the corrected twist angle:

$$F = K_{\text{tor}}\theta_{\text{corrected}}, \qquad (2)$$

where K_{tor} is the calibrated torsion constant. At short separations, the conducting spheres become polarized and no longer behave as ideal point charges. Lekner explains treating one sphere as a source charge and the other as a polarized conductor gives

$$\vec{E} = \frac{kQ}{R^2}, \qquad (3)$$

$$\vec{p} = \frac{Qa^3}{R^2}\hat{r}, \qquad (4)[3]$$

where a is the sphere radius and R is the center-to-center separation. The interaction's energy is

$$U = \frac{kQ^2}{R} - \vec{p}\cdot\vec{E} = \frac{kQ^2}{R} - \frac{kQ^2a^3}{R^4}. \qquad (4)$$

Differentiating the energy gives the corrected force:

$$F = -\frac{dU}{dR} = \frac{kQ^2}{R^2}\left[1 - 4\left(\frac{a}{R}\right)^3\right]. \qquad (5)$$

Therefore, the finite-sphere correction factor is

$$B = 1 - \frac{4a^3}{R^3}, (6)$$

$$\theta_{\text{corrected}} = \frac{\theta}{B}. \qquad (8)$$

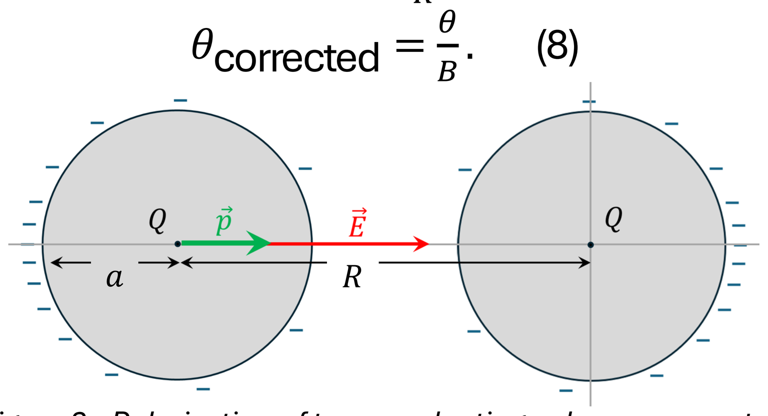

Figure 2 - Polarization of two conducting spheres separated by distance R, showing the induced dipole moment $\vec{p}$.

3 SETUP AND PROCEDURE

Models Used

- PASCO Scientific Model ES-9070 Coulomb Balance
- KEITHLEY 247 High Voltage Supply
- Faraday Ice Pail
- PASCO Scientific CI-6555 Charge Sensor
- PASCO Scientific 850 Universal Interface
- Fluke High Voltage Probe

Setup

The Coulomb balance was assembled and zeroed according to the PASCO Scientific Model ES-9070 manual [4]. A plexiglass enclosure was placed around the apparatus to reduce air currents. The charge sensor was connected to the Faraday Ice Pail to measure the charge on the conductive spheres. All trials were conducted at 3000 V rather than the manual's specified 6000 V. The power supply output was verified using a Fluke High Voltage Probe and digital multimeter.

Figure 3. Experimental setup. The charge-measurement system is shown on the left, the PASCO Scientific Model ES-9070 Coulomb Balance is enclosed at the center, and the KEITHLEY 247 High Voltage Supply is positioned on the right.

Procedure

1. The torsion wire was calibrated using 20, 40, 50, and 70 mg masses. The torsion constant was calculated for each mass and averaged. The apparatus was then re-zeroed and positioned vertically.
2. Both conductive spheres were charged to 3000 V. The initial sphere charge and room humidity were recorded.
3. The equilibrium twist angle was measured at sphere separations of 20, 14, 10, 9, 8, 7, 6, and 5 cm.
5. After each trial, the remaining sphere charge was recorded and both spheres were grounded.
6. Trials were repeated until the measured angles agreed within ±1°. The largest qualifying angle at each distance was used for analysis.
6. Measurements were converted to SI units, and the finite-sphere correction from the Model ES-9070 manual was applied.
7. Corrected and uncorrected data were analyzed using log-log and inverse-square plots with linear fits.
8. One corrected trial was used to calculate the electrostatic force and the experimental value of Coulomb's constant, (k).

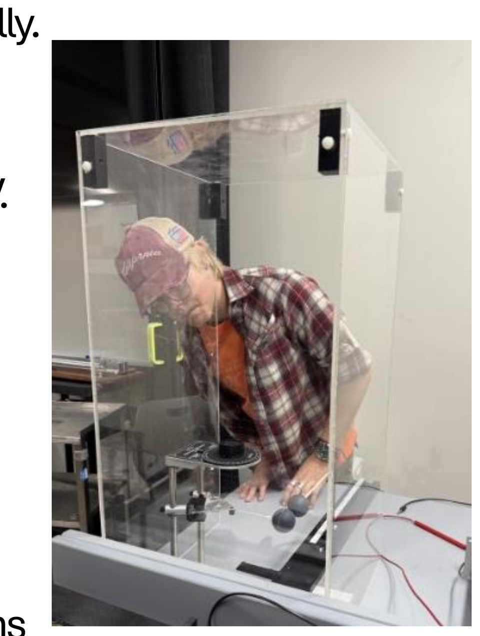

Figure 4. Data collection using the enclosed Coulomb Balance

4 RESULTS

Torsion-Wire Calibration

Known masses were used to calibrate the torsion wire. Dividing the gravitational force of each mass by its measured twist angle produced an average torsion constant of:

$$K_{\text{tor}} = 1.43 \times 10^{-6}\ \text{N/degree}$$

This value was used to convert measured twist angles into electrostatic force.

Table 1. Torsion-wire calibration results.

Mass (g)	Twist Angle (degrees)	Weight (N)	K_{tor} (N/degress)
0.00002	140	1.96E-4	1.40E-6
0.00004	275	3.92E-4	1.43E-6
0.00005	335	4.90E-4	1.46E-6
0.00007	474	6.86E-4	1.45E-6

Force vs Distance

Measurements were collected at eight sphere separations under a constant relative humidity of 51%. Twist angle increased as separation decreased, indicating a greater electrostatic force at shorter distances. The finite-sphere correction increased the measured angles, particularly at the smallest separations.

Table 2. Measured and corrected force-versus-distance data.

Distance R (m)	Twist Angle (degrees)	Charge before (C)	Charge after (C)	Humidity (%)
0.20	3	5.75E-09	4.63E-09	51
0.14	6	5.84E-09	4.48E-09	
0.10	8	5.80E-09	4.35E-09	
0.09	11	5.82E-09	4.88E-09	
0.08	15	5.78E-09	4.45E-09	
0.07	21	5.88E-09	4.50E-09	
0.06	26	5.86E-09	4.46E-09	
0.05	32	5.88E-09	4.42E-09	

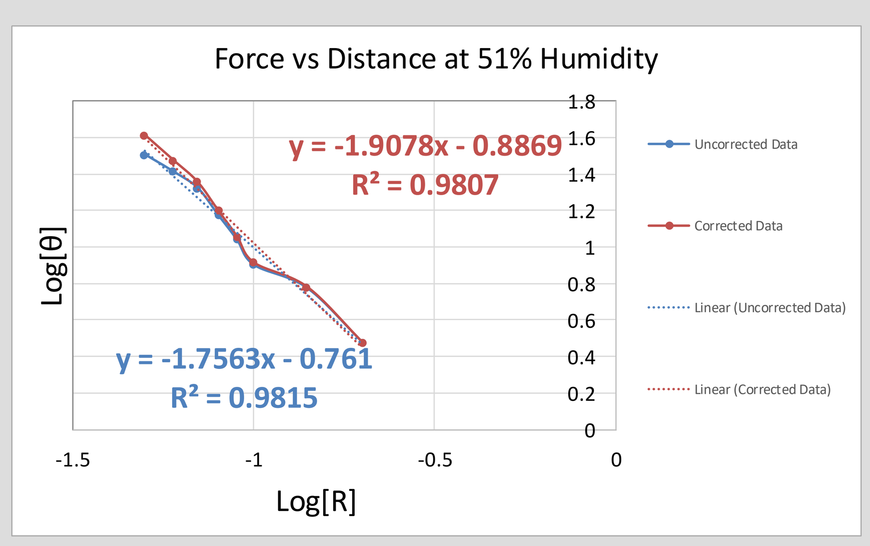

Figure 5. Log–log plot of corrected and uncorrected twist angle versus sphere separation, including linear trendlines.

The log–log analysis produced a corrected slope of −1.91, close to the ideal inverse-square slope of −2. The corrected fit had $R^2 = 0.9807$, while the uncorrected data produced a slope of −1.76 with $R^2 = 0.9815$. The finite-sphere correction therefore improved agreement with Coulomb's law.

Calculating Coulombs Constant

Coulomb's constant was calculated from complete data sets and from additional individual trials conducted specifically to improve measurement accuracy. The best individual trial yielded

$$k = 7.3 \times 10^9$$

approximately 19% below the accepted value of 8.99×10^9 . This trial had an initial charge of 3.7×10^{-9} C, a final charge of 3.0×10^{-9} C, a corrected twist angle of 12°, and a sphere separation of 0.07 m.

5 CHALLENGES

- High summer humidity was the main challenge. Relative humidity reached 64%, causing rapid charge loss and delaying data collection by about one week. Published surface-charge measurements have likewise shown faster charge decay at higher relative humidity, supporting humidity as a likely contributor to charge loss in our experiment [5].
- Potential charge loss while moving the sphere
- Nearby charged objects applying a force
- Floor vibrations, and air currents affecting the pendulum

Additional Finding: Humidity-Dependent Charge Decay

As a dehumidifier gradually reduced the room's humidity, charge decay was measured at several relative-humidity levels. These observations produced the empirical model

$$Q(t) = Q_0 e^{\frac{-t}{2156 - 32 \times RH}}$$

where t is time in seconds and RH is relative humidity expressed as a percentage. The model estimates the remaining charge from its initial value, elapsed time, and humidity.

6 Future Work

We hypothesize that our systematically low values of k are related to humidity. The current trendline predicts the accepted value near 36% relative humidity. Future trials should be conducted as seasonal humidity decreases, particularly near 36% RH, to determine whether drier conditions improve accuracy.

A second experiment could measure the magnetic permeability of free space, μ_0. Our experimental Coulomb constant would then be used to calculate the permittivity of free space,

$$\varepsilon_0 = \frac{1}{4\pi k}.$$

The experimental values of μ_0 and ε_0 could then be combined to estimate the speed of light:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}.$$

REFERENCES

1. R. Williams, "June 1785: Coulomb measures the electric force," APS News 25(6), 3 (2016).
2. D. J. Griffiths, Introduction to Electrodynamics, 4th ed. (Pearson, Boston, MA, 2013), pp. xv and 60.
3. J. Lekner, Electrostatics of Conducting Cylinders and Spheres (AIP Publishing, Melville, NY, 2021), Sec. 1.2.1.
4. PASCO Scientific, Coulomb Balance: Instruction Manual and Experiment Guide for the PASCO Scientific Model ES-9070, Manual No. 012-03760E (PASCO Scientific, Roseville, CA, 1999).
5. M. Navarro-Rodríguez, E. Palacios-Lidón, and A. M. Somoza, "The surface charge decay: A theoretical and experimental analysis," Appl. Surf. Sci. 610, 155437 (2023).