

Introduction

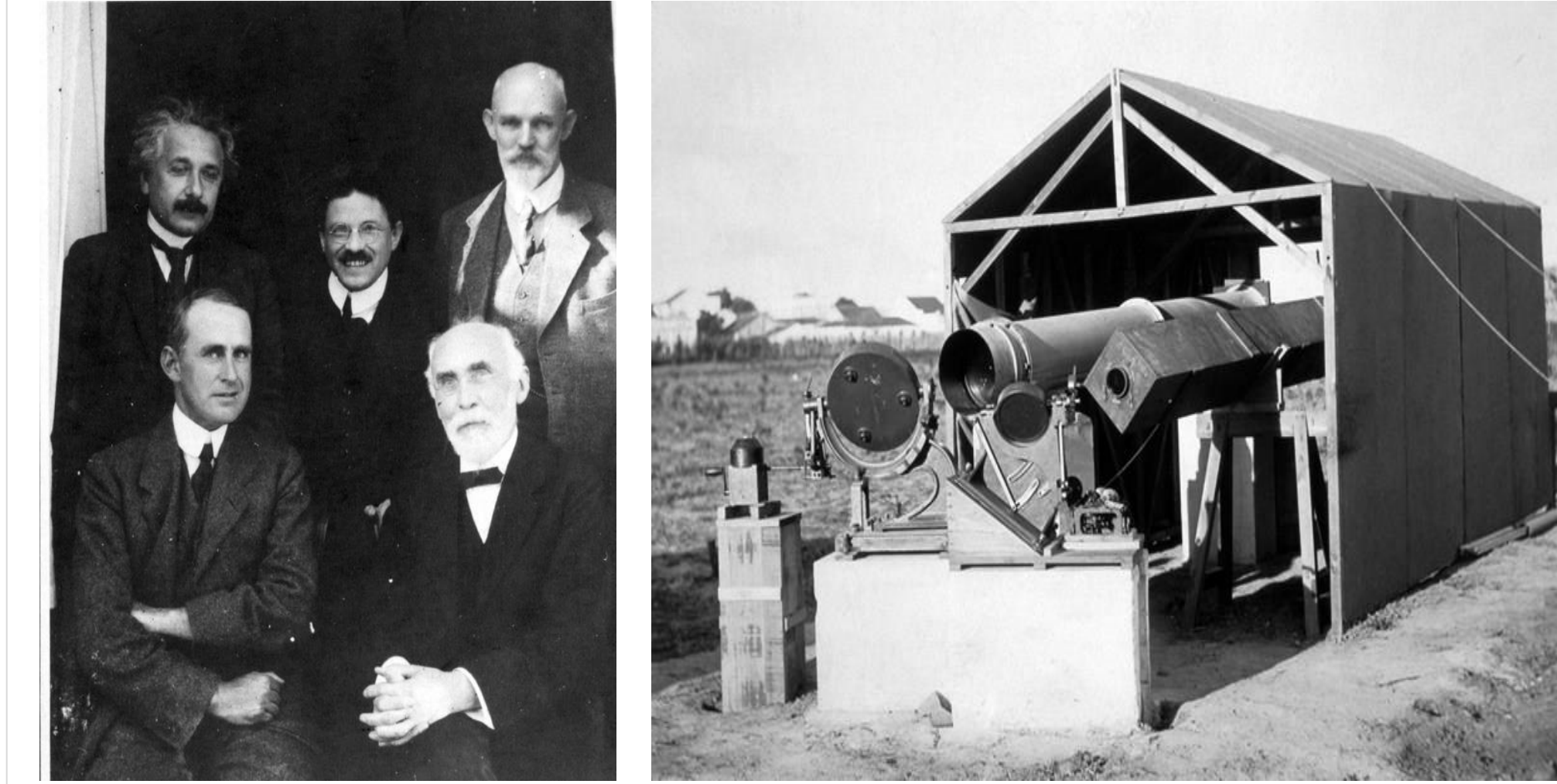


Fig. 1a (left): Einstein (top left) and Eddington (bottom left) in 1923; Fig. 1b (right): equipment used by Eddington to observe the solar eclipse of 1919.

Among the most important events in the history of science was the overthrow of Newton's theory of gravitation by Albert Einstein's theory of General Relativity. This overthrow was accomplished almost overnight owing to observations made by the English astrophysicist Arthur Eddington and his colleagues during the total solar eclipse of May, 1919 [1-3]. To make them, they had to travel by steamship to the path of totality in Brazil and West Africa, carrying large amounts of heavy equipment such as 16-inch telescopes and photographic plates (Fig. 1). When their results were announced in November of that year, the front page of the *New York Times* read: "LIGHTS ALL ASKEW IN THE HEAVENS! ... EINSTEIN THEORY TRIUMPHS!"

In Einstein's theory, gravity is not a force acting through space between massive bodies (as in Newton's theory). Rather, it is a manifestation of the *shape of space* around those bodies. The Sun's mass warps space, bending light rays from background stars so that they appear to be deflected away from the Sun as seen from Earth (Fig. 2). The effect is

only visible during a total solar eclipse, when the Sun passes behind the Moon, because the stars are otherwise lost in the glare of the Sun. Space curvature actually produces only one-half of the total deflection; the other half is explained by the Equivalence Principle, which states that the effects of gravity are equivalent to those of acceleration. If you shoot a light beam across an elevator car that is accelerating upward, the beam hits the opposite wall at a lower point. This looks like a curved path to someone in the elevator. Hence, since gravitation is equivalent to acceleration, light must follow the same curved path in a gravitational field. The total deflection due to *both* effects together is

$$\Delta\theta = \frac{4GM_{\odot}}{c^2 R} = \frac{1.7''}{R/R_{\odot}}, \quad (1)$$

where G is Newton's gravitational constant, c is the speed of light, $M_{\odot}$ and $R_{\odot}$ are the mass and radius of the Sun, R is the distance from the Sun, and we convert from radians to arcseconds (") via $1 \text{ rad} = 206,265''$.

Procedure

We chose the 2023 total solar eclipse in Exmouth due to the position of the sun having not two but three stars within two solar radii on opposing sides. While one of these stars, HIP 8588, would have a magnitude of 5.9, and thus would be easily visible, the other two, GAIA DR3 and HIP 8327 were 9th and 8th magnitude stars, far less bright. These stars would be denoted as Stars A, B and C respectively. Each should be shifted away from the Sun by 1.2" during totality according to Eq.(1), so their separation should increase by 2.4" for Stars A and B and A and C. B and C, being on the same side, should not have a measurable change in separation at all, thus serving as a null test.

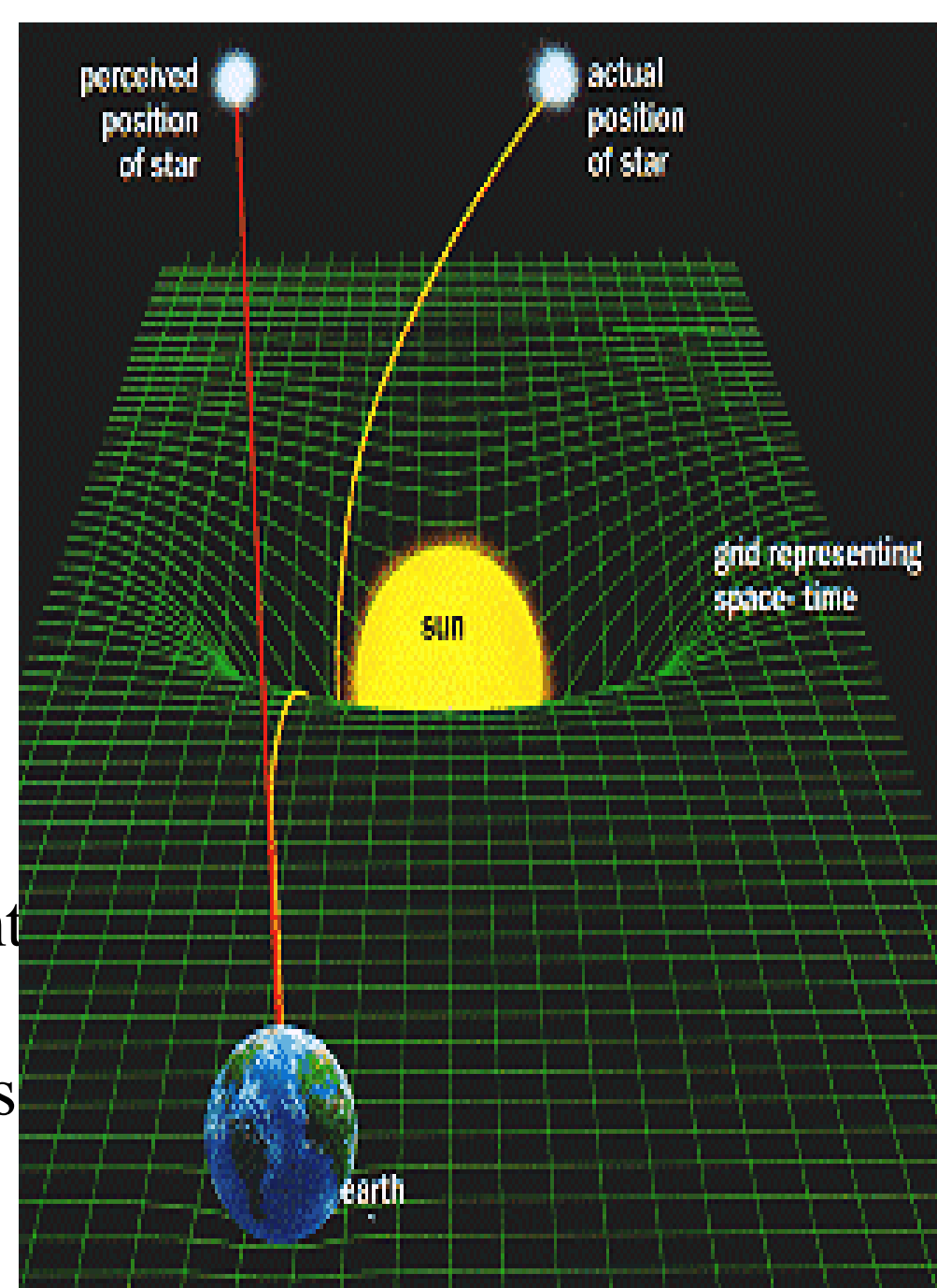
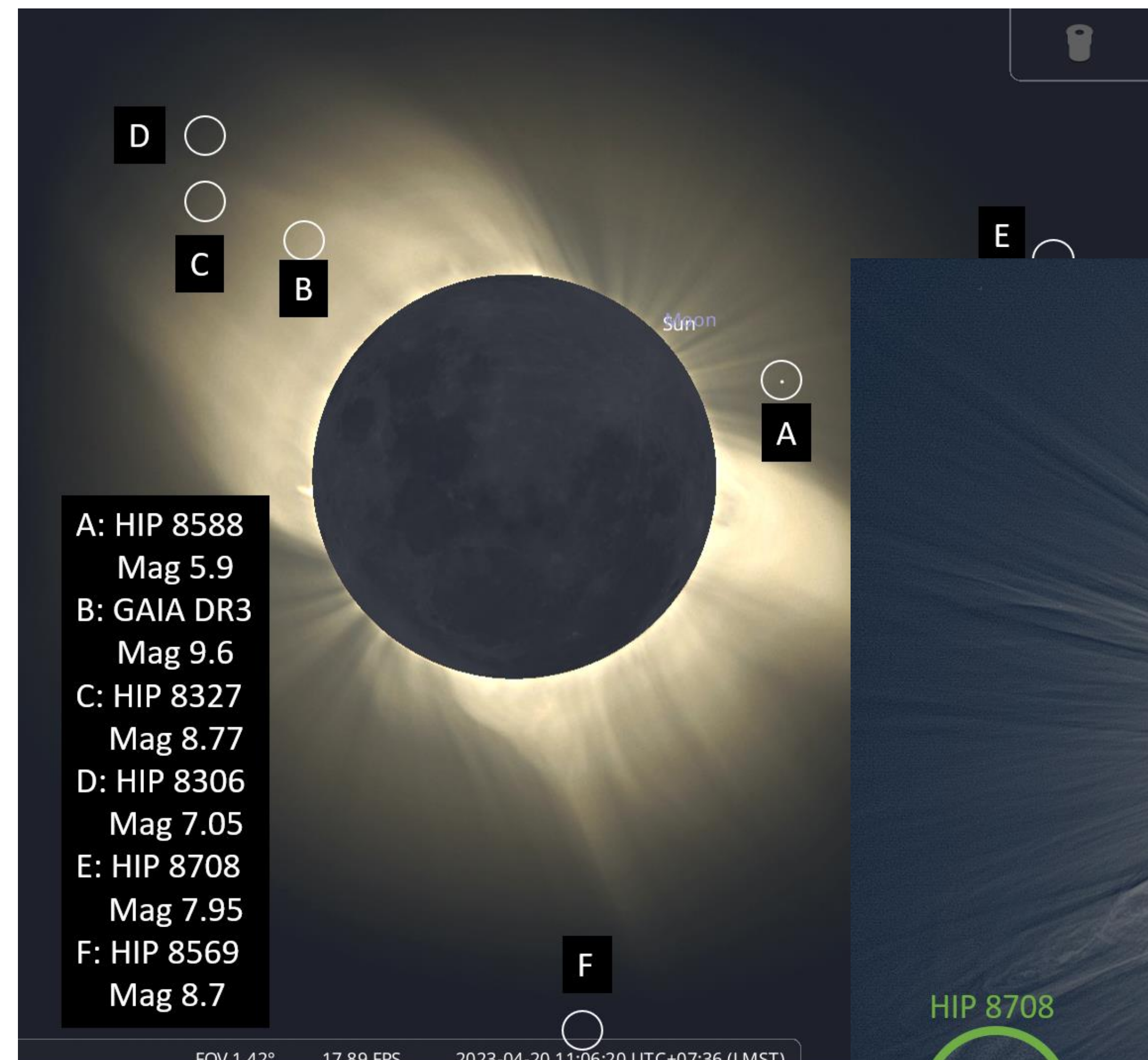


Fig. 2: Schematic depiction of light deflection due to curved space.



Thus, our strategy would be to use the other denoted reference stars, Stars D, E and F, to establish a scale and use this scale to calculate the angular positions and distance between A and B and A and during totality. We would then compare this to the true angular distance between the two stars (as looked up in the SIMBAD

database) and check if the difference matched Eq. (1).

Our data came from Stellarium software (which we used to simulate the eclipse and identify our target and reference stars, Fig. 3a) plus a composite of 77 high-resolution images of the actual eclipse taken by Miroslav Druckmuller (Fig. 3b [4]). We were able to find all three target stars (A,B,C; red insets) and the three reference stars from Fig. 3A: HIP 8306 ("Star D"), HIP 8708 ("Star E") and HIP 8569 ("Star F"; green insets).

The next step was to use linear algebra. Our measured positions x, y are in pixels. To convert to RA and Dec (α, δ), we transformed via

$$\begin{aligned} \alpha &= \alpha_0 + \beta(x \cos \theta + y \sin \theta), \\ \delta &= \delta_0 + \beta(-x \sin \theta + y \cos \theta), \end{aligned} \quad (2)$$

where (α_0, β_0) is the location of the image origin (in deg), β is a rescaling factor (in deg/px), and θ is a rotation angle (Fig. 4). We can solve for all 4 unknowns by inverting Eqs. (2) for any 2 reference stars with measured positions (x, y) in the image and known coordinates (α, β) in the sky. We did this for all three pairs of reference stars (DE, EF, DF), finding average values $\alpha_0 = 26.41 \pm 0.04^\circ$, $\delta_0 = 10.76 \pm 0.26^\circ$, $\beta = 0.00074 \pm 0.00011^\circ/\text{px}$ and $\theta = 7.01 \pm 7.52^\circ$. Further work needs to be done to bring down the uncertainty in θ . Putting these values back into Eqs. (2), we found the locations of Stars A, B and C in our image as follows:

$$\begin{aligned} \alpha_A &= 27.823 \pm 0.12^\circ, & \alpha_B &= 27.048 \pm 0.05^\circ, & \alpha_C &= 27.023 \pm 0.04^\circ, \\ \delta_A &= 11.049 \pm 0.09^\circ, & \delta_B &= 11.130 \pm 0.15^\circ, & \delta_C &= 11.063 \pm 0.16^\circ, \end{aligned} \quad (3)$$

where uncertainties come from the standard deviations from all three reference stars.

The angular separation between stars could then be found from the fundamental formula of spherical trigonometry [5,6]:

$$\theta = \cos^{-1}[\sin \delta_A \sin \delta_B + \cos \delta_A \cos \delta_B \cos(\alpha_A - \alpha_B)]$$

The upper and lower limits were derived by combining the upper and lower limits on (α, δ) from Eq. (3).

Results

We find an average angular distance between Stars A,B of $2755 \pm 433''$ which is reasonable; however our calculated "true" angular distance is much too small at 1529" so not have a result to

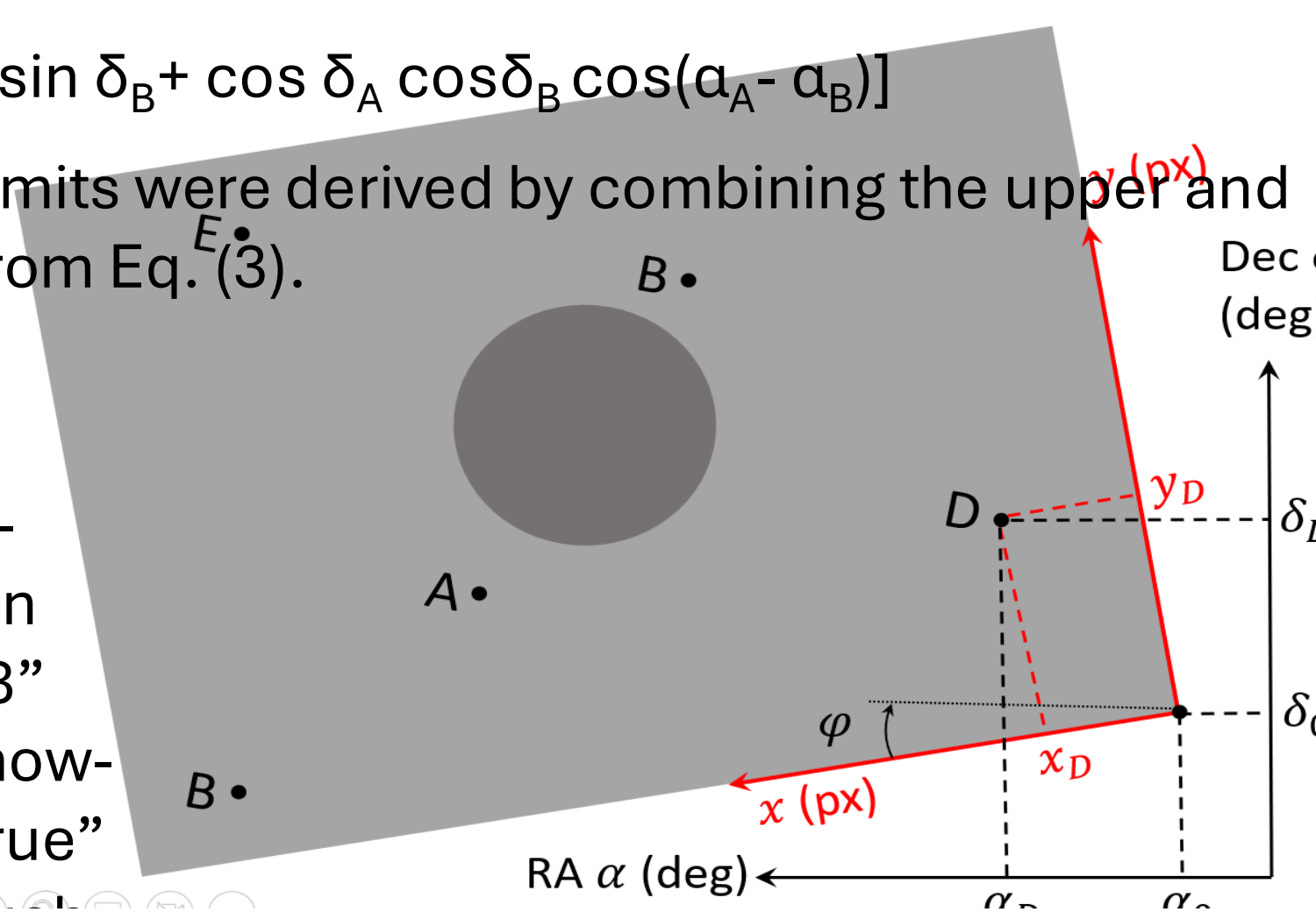


Fig. 2: Setup for transforming the location of Star D from image coordinates (x,y) in pixels to equatorial coordinates (α, δ) in degrees.

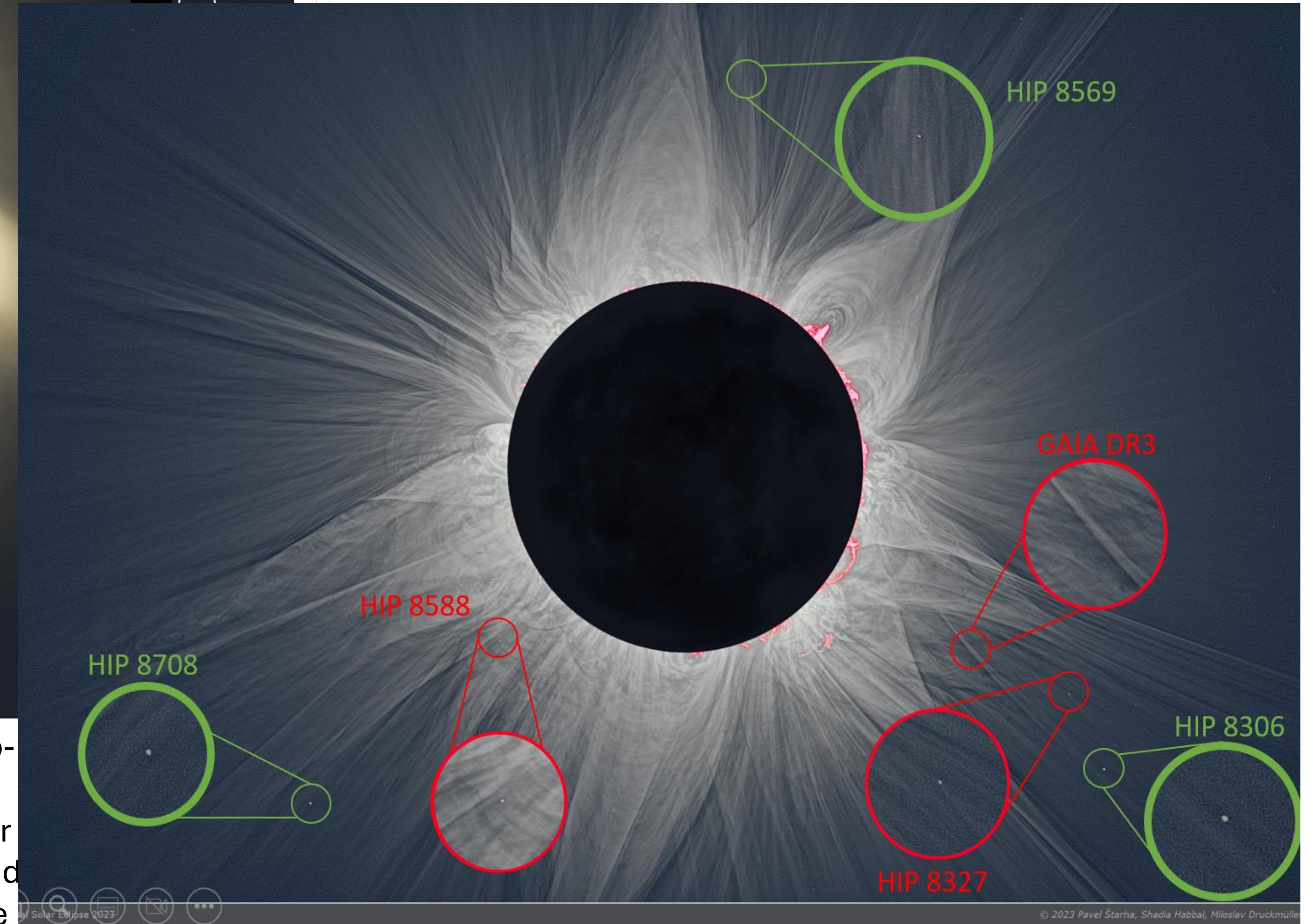


Fig. 3a (left): 2023 Exmouth eclipse viewed through *Stellarium* web planetarium; Fig 3b(right): Same frame of reference displayed in Druckmuller high definition photography. Stars used and reference stars both displayed in magnified circles.

Future Work

For further work, we would like to extend this method to other total solar eclipses such as the upcoming August 12, 2026 eclipse in Iceland. A *Stellarium* simulation shows an 8th magnitude star and a 9th magnitude star, HIP 46339 and HIP 46318 respectively, close enough to be deflected by at least 1" (but will not be on opposing sides of the sun, so the angular distance will be less than 2"), and a 9th magnitude star, HD 82127, just barely possibly within distance to be deflected on the opposing side. There is a dearth of bright reference stars during this time period, but at least two 6th and 7th magnitude stars will be nearby enough to provide a reference scale. Further historical eclipses via historic photography from Dr. Druckmuller [4] and others could be used to analyze past eclipses as well. A major goal would be to prove the validity of this method, seeing as how a previous attempt in 2017 [5] obtained a measurement of 4.1" with an uncertainty of 14.7" out of the expected 2.4 ". Further trials with more eclipses would provide more sources of data to further reduce the uncertainty, ultimately aiming to either definitively show that this method works, and aim to explain the inconsistency of the results.

Acknowledgments

We thank the Maryland Space Grant Consortium and Fisher College of Science and Mathematics at Towson University for their support of this project.

References

- [1] Dyson, F.W., Eddington, A.S. and Davidson, C., "A determination of the deflection of light by the Sun's gravitational field, from observations made at the total eclipse of May 29, 1919," *Phil. Trans. R. Soc. London A* 220 (1920) pp. 291-333
- [2] Coles, P., "Einstein, Eddington and the 1919 Eclipse," in V.J. Martínez, V. Trimble and M.J. Pons-Bordería (eds) *Historical Development of Modern Cosmology* (San Francisco: ASP Conf. Proc. Vol. 252, 2001) pp. 21-41
- [3] Kennefick, D., "Testing relativity from the 1919 eclipse—a question of bias," *Physics Today* 62 (2009) 27
- [4] Druckmuller, "A new numerical method of total solar eclipse photography processing," *Contributions of the Astronomical Observatory Skalnaté Pleso* (2006)
- [5] Overduin et al, "Solar Eclipses as a Teaching Opportunity in Relativity," *The Physics Educator* (2020)