Magnetic Perpetual Motion?

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Introduction

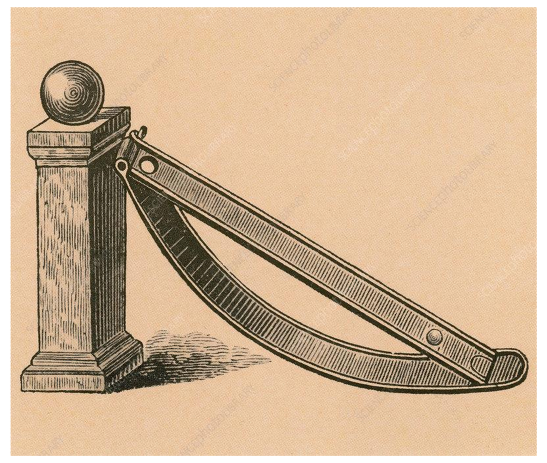


Fig. 1: Maricourt's magnetic *perpetuum mobile* of 1269 [1], as illustrated in a book by Bishop John Wilkins in 1691 [2]

Proponents of perpetual motion have long been captivated by magnets, presumably because magnetic attraction seems to offer an infinite source of invisible and hence apparently "free" work. A simple example was proposed by medieval physician Pierre de Maricourt in 1269. Maricourt's book was plagiarized by the Jesuit scholar Jean Taisner in 1572 [1] and the idea was popularized by English bishop John Wilkins in 1691 (Fig. 1), so it is sometimes known as the "Taisnerius engine" or "Wilkins ramp" [2]. A permanent magnet pulls an iron ball up a ramp. At the top of the ramp, it falls through a hole to the bottom of the ramp and repeats the loop, *ad infinitum*. Of course, due to friction and demagnetization, this motion could never continue perpetually (Second Law of Thermodynamics). But could it even make one or more complete circuits? A youtube clip purports to show a real-life Taisnerius engine making many successful round trips [3]. Is it a hoax, or digital animation? We decided to investigate the question.

Theory

The situation is depicted in Fig. 3. One can think of at least two dynamical (force) requirements: (1) At the top, gravity must be stronger than the upward component of magnetic force, and (2) at the bottom, magnetic force must be stronger than the component of gravity along the ramp. In addition, one could apply an energy requirement: (3) the gain in gravitational potential energy mgh as the ball climbs the ramp must exceed its drop in magnetic potential energy ΔU_m as it approaches the magnet. Mathematically, we could write:

$$mg > F_m(d)\sin\theta \quad (1)$$

$$F_m(nd) > mg\sin\theta \quad (2)$$

and $mgh > \Delta U_m$, or (using $\Delta U = -W = -\int \vec{F}\cdot\vec{d\ell}$):

$$(n-1)mgd\sin\theta > -\int_{nd}^{d} F_m(z)dz \quad (3)$$

Fig. 2: Video clip of a supposed working model undergoing many complete loops on the internet [2]

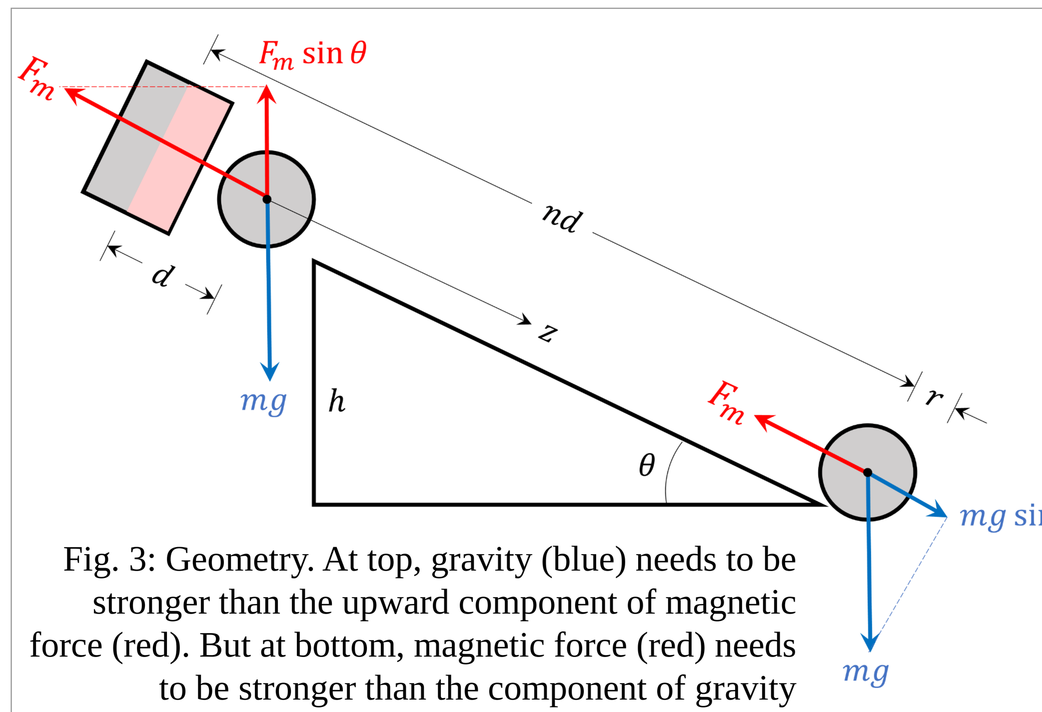


Fig. 3: Geometry. At top, gravity (blue) needs to be stronger than the upward component of magnetic force (red). But at bottom, magnetic force (red) needs to be stronger than the component of gravity downward along the ramp (blue).

To evaluate the constraints (1)-(3), we need an expression for the force $F_m(z)$ of a permanent magnet on a ferro-magnetic (steel/iron) sphere, where z is distance from the center of the magnet along its symmetry axis (Fig. 3).

In general, force is the gradient of potential, so $\vec{F} = -\vec{\nabla}U$ or $F = -dU/dz$ in this case. The ball acts like a compass needle (a magnetic dipole) with a moment $\vec{p}$ in the external field $\vec{B}$ of the magnet. The potential energy is $U = -\vec{p}\cdot\vec{B}$. The dipole moment of a solid sphere in a field that is close to uniform (i.e., on scales comparable to radius r) is [4]

$$\vec{p} = \left(\frac{\mu-1}{\mu+2}\right) r^3\vec{B} \approx r^3\vec{B} \quad (4)$$

for strongly ferromagnetic materials like iron, with $\mu \gg 1$. Hence potential $U = -r^3\vec{B}\cdot\vec{B} = -r^3B^2$. The fact that B is squared is important; it arises because both the external field *and* induced dipole contribute to the interaction energy. At distances greater than the magnet size, the field is that of a dipole so $B(z) \propto z^{-3}$ along the axis. Hence we expect:

$$F_m(z) = kz^{-p} \quad (5)$$

with k = const and p =7. This is much steeper than the Newtonian force between masses or the Coulomb force between charges. It imposes severe limitations on our design. If the magnet attracts the ball with a force of 0.01 mN at a distance of 10 cm, for example, then $k = 10^{-12}$ in SI units so the force at 1 cm will be 100 N. It is difficult to imagine changing the setup in such a way as to weaken this force at the top, while still keeping it strong enough to attract the ball up the ramp at the bottom.

Fig. 4: using a smartphone to record the motion of the ball in slow motion so that its position, speed and acceleration can be determined with a Tracker app

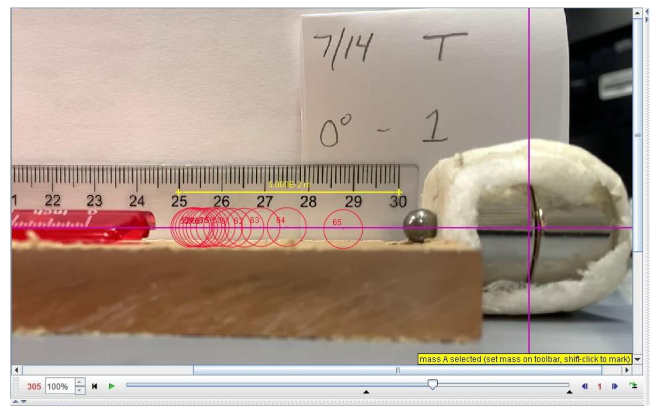


Fig. 5: the rotoscoping process. Tracker measures position as a function of time, from which we derive speed and acceleration

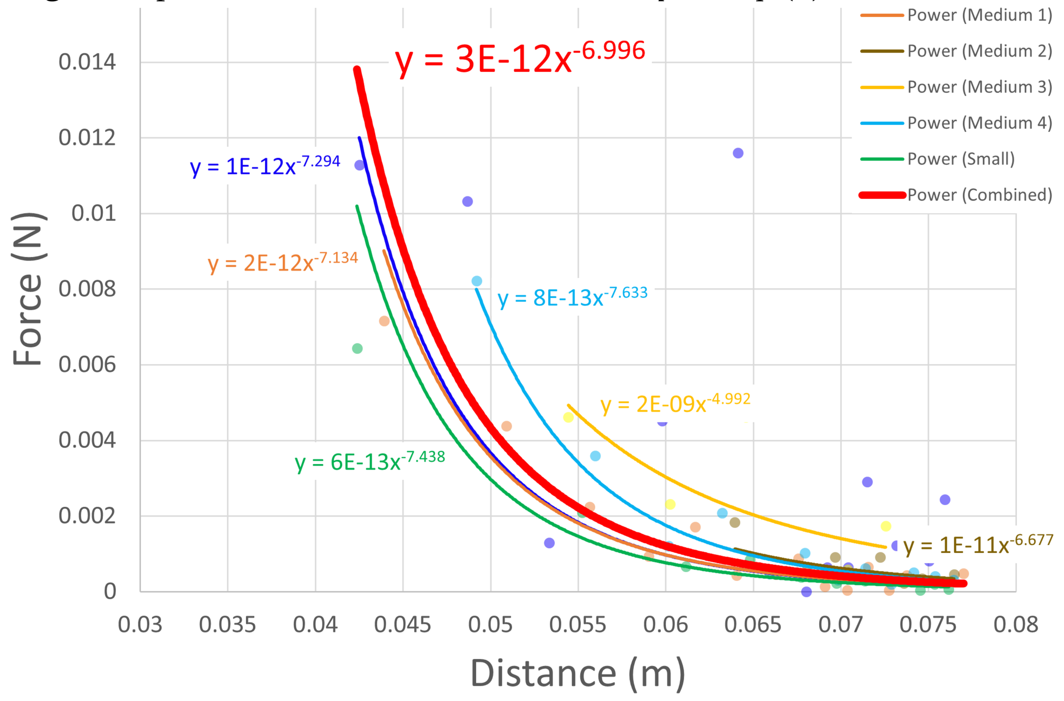
Experiment

Because this force law is so critical, we tested it experimentally using a neodymium disk magnet, three ferro-magnetic steel balls of radii 3.2 mm, 4.8 mm and 6.4 mm, and a smooth plexiglass ramp with a slight groove. First, we used a smartphone to make high-speed videorecordings of the ball being pulled towards the magnet at 240 frames per second (Fig. 4). We then used the free software package Tracker to manually rotoscope position vs. time data on a computer (Fig. 5). Finally, we used Excel to derive speed and acceleration from these position data via $v = \Delta z/\Delta t$ and $a = \Delta v/\Delta t$, and calculated $F = ma$ with m the mass of the ball. We could then plot F as a function of z and fit the data to Eq. (5) to determine the best-fit values of k and p.

Results are shown in Fig. 6, which confirms our theoretical expectations: $p = 7.0$ and $k = 3 \times 10^{-12}$ Nm^7. Two caveats apply: first, these values depend somewhat on the range of data considered. We rejected data beyond a "cutoff distance" of 7.7 cm from the magnet. Adjusting this cutoff (within the range $7.4 - 8.1$ cm) produces a broader range of best-fit values, $p = 7.7 \pm 0.8$. Smaller cutoffs do not leave us with enough data, while longer ones take us into the non-magnetic regime where the signal is dominated by noise.

Second, our use of $F = ma$ ignores the effects of friction and rolling. If the ball rolls without slipping, then a given acceleration a implies a larger force F_m, because the magnet must not only pull the magnet but also overcome static friction $f_s = \mu_s mg$ with μ_s =0.4-0.5 for steel on plexiglass and $g = 9.8$ m/s^2. Newton's Second Law then gives $F_m = m(a + \mu_s g) \approx \mu_s mg$ since $\mu_s g \gg a$ over most of the range. Our data are inconsistent with this and show strong dependence on distance, implying that the effects of friction and rolling are relatively unimportant.

Fig. 6: Experimental best-fit values of k and p in Eq. (5)



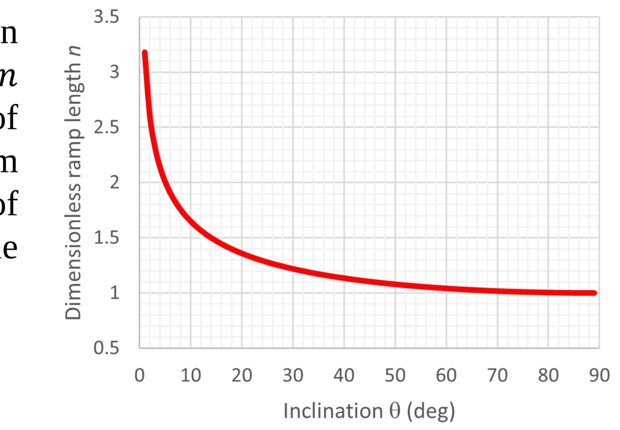


Fig. 7: upper limits on dimensionless ramp length n (the ratio of the distances of the magnet from the bottom to the top) as a function of inclination angle

Conclusions

Putting our force law (5) into Eqs. (1) and (2), we obtain the force constraints:

$$mg > kd^{-p}\sin\theta \quad (6)$$

$$k(nd)^{-p} > mg\sin\theta \quad (7)$$

Combining these two results and eliminating the ratio of forces kd^{-p}/mg, we obtain a constraint on the ramp length n in terms of inclination θ only:

$$n^p\sin^2\theta < 1 \quad (8)$$

With $p = 7$, this bound places an extremely tight constraint on the ramp (Fig. 7): for any realistic inclination (> 10 deg) the bottom can be no farther than 1.6 times farther from the magnet than the top! It is hard to see how such a ramp could work in practice. The reason boils down to the high value of p, which restricts the range of distances over which the strength of magnetic and gravitational forces can change appreciably relative to each other.

Putting Eq. (5) into Eq. (3) and integrating gives a third constraint based on energy. With $p = 7$, this imposes a lower limit on inclination:

$$\sin\theta > \frac{n(n^6-1)k(nd)^{-7}}{6(n-1)mg} \quad (9)$$

At the point of closest approach to the magnet, there must be enough potential energy left to convert into kinetic energy and return the ball to its starting point.

Combining Eq. (9) with Eq. (7) and simplifying, we obtain a constraint on *ramp length alone:*

$$6(n-1) > n(n^6-1) \quad (10)$$

Eq. (10) has no positive real solutions (other than the trivial one, $n = 1$, which means a ramp of zero length). This suggests that it is impossible to satisfy both the force and energy constraints, even for a single circuit.

Acknowledgment

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References

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[2] Wilkins, J., *Mathematical Magick: or The Wonders that may be Performed with Mechanical Geometry*, 2d ed. (London: Ric. Baldwin,1691)

[3] Ivanov, Valery, "John Wilkins Ramp, Perpetual Motion Machine," https://www.youtube.com/watch?v=V70w3cxDJIM (2016, accessed July 27, 2022)

[4] Jackson, J.D. *Classical Electrodynamics*, 2d ed. (New York: John Wiley & Sons, 1975), Eqs. (5.107) and (5.115)