Rolling Railgun: A Lab Activity for Introductory Electromagnetism

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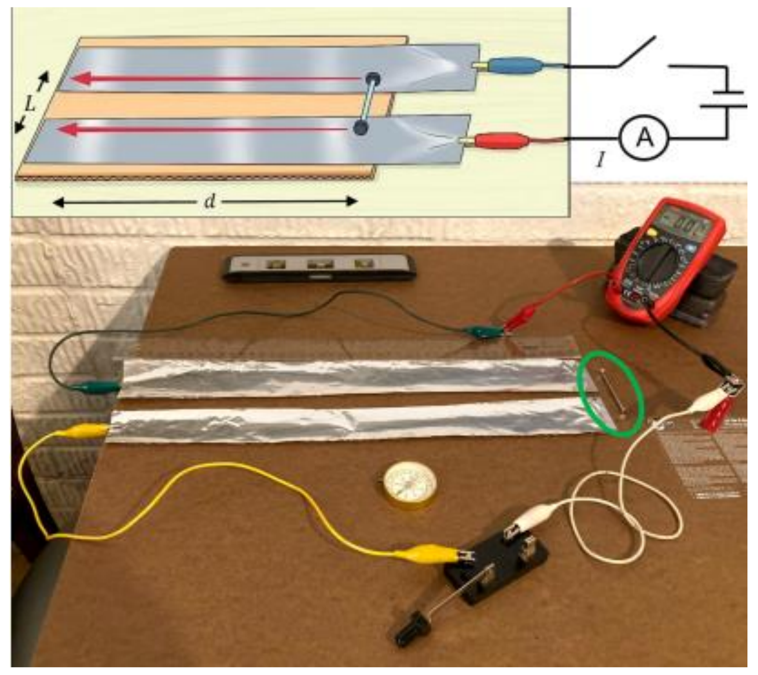


Fig. 1: A diagram (top) and photograph of a student setup (bottom) showing required components. The projectile (two neodymium coin magnet wheels connected by a ferromagnetic conducting axle) is circled in green.

Introduction

In Physics II (electromagnetism), students often struggle because the most important elements of the subject are more abstract than the forces and bodies in Physics I. To address this, we devised a lab activity in which students experiment and build their own "rolling railgun", then attempt to explain its behavior both qualitatively and quantitatively. Railguns are mentioned in some textbooks but have not received much attention in the physics education community [1-5]. We designed our activity to be "pandemic-friendly": all required components can be mailed to students at home.

Qualitative and Quantitative Experiment

Students begin by laying aluminum strips on a level, insulating work surface about 2 cm apart, and smooth them down as flat as possible (Fig. 1). They connect the strips to the battery with a switch to turn the voltage between the rails on and off, and an ammeter to monitor the current. To form the projectile, students center the magnets on both ends of the wire axle, ensuring that like poles face each other. They then place the projectile in the middle of the rails and close the switch. It should accelerate one way or the other. Their task is to explain this acceleration using their knowledge of the Lorentz force law and the left-hand rule. The goal is for them to find their way to something like the diagrams in Fig. 2.

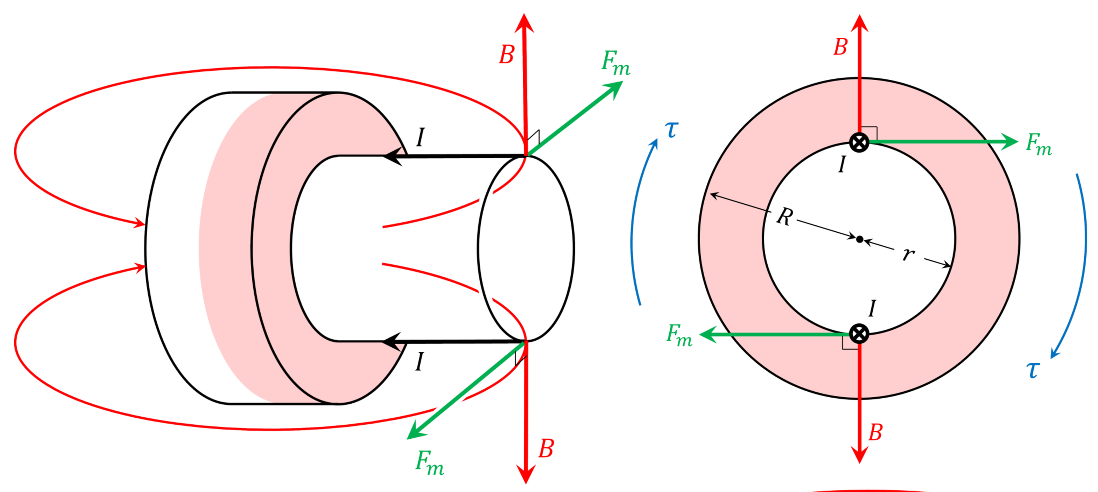
The quantitative aspect of this experiment is for students to measure their railgun's acceleration and hence obtain a value for the field strength B of the magnets which can be compared to the manufacturer's factory specifications. Taking $\vec{B}$ to be approximately perpendicular to the current in the axle, Newton's second law with the Lorentz force $\vec{F}_m = I\vec{L} \times \vec{B}$ gives

$$F_m = 2nILB = M_p a \ , \qquad (1)$$

for n pairs of magnets, where I is current, L is axle length, m and M are the masses of the axle and magnet respectively, and $M_p = m + 2nM$. Using the kinematic formula of $d = d_0 + v_0 t + \frac{1}{2}at^2$ and re-arranging gives

$$B = \frac{M_p d}{nILt^2} \ . \qquad (2)$$

Careful students will notice that the Lorentz forces on opposite sides of the axle point in opposite directions. The projectile is accelerated along the rails by *torque*, not force (Fig. 2).



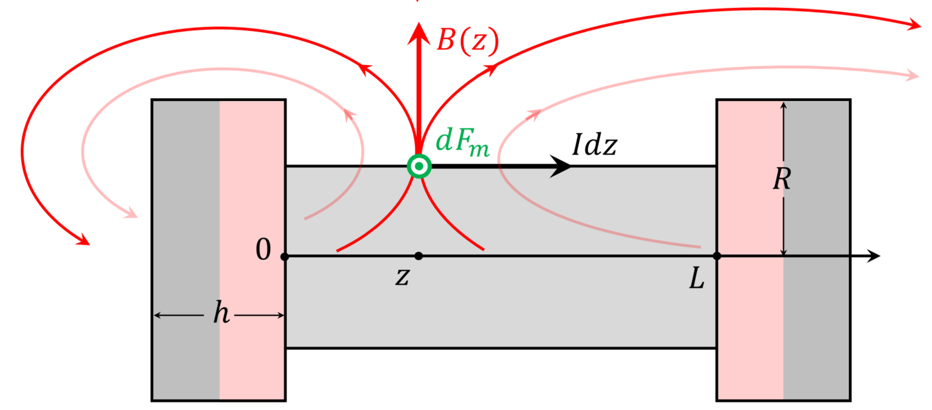


Fig.2: (top) Three-quarter view (left) and edge-on view (right) of the situation.
Fig.3: Side view of the projectile showing the Lorentz force dF_m on the current element Idz

With this insight, students can replace Eq. (1) with the angular form of Newton's second law:

$$\tau = |r \times F_m| = rF_m = I_m \alpha = \frac{I_m a}{R} \ . \qquad (3)$$

where I_m is moment of inertia, α is angular acceleration, and r and R are the radii of the axle and magnets respectively. Since these are solid cylinders, their combined moment of inertia is $I_m = \frac{1}{2}MR^2 + 2n \times \frac{1}{2}mr^2$. Putting these equations together, students obtain a modified version of Eq. (2):

$$B = \frac{M_{\text{eff}} d}{nILt^2} \ , \qquad (4)$$

where $M_{\text{eff}} = (r/2R)m + (R/2r)2nM$ is the effective "rotational mass." With $n = 2$, $d = 38 \pm 1$ cm, $L = 6.3 \pm 0.1$ cm, $m = 3.0 \pm 0.2$ g, $r = 1.4 \pm 0.1$ mm, $M = 0.38 \pm 0.05$ g and $R = 3.2 \pm 0.1$ mm, we measured $I = 0.58 \pm 0.19$ A and $t = 1.4 \pm 0.2$ s, implying field strengths $B = 120 \pm 50$ G from Eq. (2) and 70 ± 20 G from Eq. (4). These values are several times smaller than the advertised strengths of the magnets, averaged over the length of the axle (of order 280 G), likely because railgun acceleration is sensitive only to the component of $\vec{B}$ perpendicular to the current in the axle.

Optimization

Students can use their calculus skills to maximize the speed of their railguns by varying the values of n, r and L. Using $m = \rho\pi r^2$ where ρ is axle density, Eq. (4) becomes

$$a = \frac{2nILBRr}{nMR^2 + \frac{\pi}{2}\rho L r^4} \ . \qquad (5)$$

Differentiating with respect to n and L, we obtain derivatives that vanish as $n \to \infty$ and $L \to \infty$, giving theoretical upper limits of $a \leq \frac{2ILBr}{MR} = 0.6$ m/s² and $a \leq \frac{4nIBR}{\pi\rho r^3} = 2.8$ m/s² respectively. Optimizing for axle radius is more interesting; we find an optimal radius $r_* = \left(\frac{2nMR^2}{3\pi\rho L}\right)^{\frac{1}{4}} = 1.3$ mm, for which predicted acceleration $a_* = \frac{3ILBr_*}{2MR} = 0.4$ m/s², consistent with our measured results.

	h (in):	1/32	1/16	1/8	1/4	3/8	1/2	5/8	3/4
	h (m):	0.00079	0.00159	0.00318	0.00635	0.00953	0.0127	0.01588	0.01905
D (in):	R (m):								
1/16	0.00079	0.305	0.396	0.45	0.471	0.472	0.468	0.462	0.455
1/8	0.00159	0.738	1.05	1.197	1.098	0.956	0.837	0.742	0.665
			0.062	0.243			0.324		
1/4	0.00318	0.784	0.868	0.77	0.544	0.409	0.326	0.27	0.23
		0.326	0.262	0.411	0.562	0.36	0.331		
3/8	0.00476	0.369	0.36	0.31	0.226	0.174	0.14	0.117	0.1
			0.4	0.689	0.281	0.459			
1/2	0.00635	0.174	0.166	0.147	0.113	0.09	0.074	0.062	0.054
			0.399	0.509	0.418	0.311	0.253		
5/8	0.00794	0.092	0.088	0.08	0.064	0.052	0.044	0.038	0.033
3/4	0.00953	0.054	0.052	0.048	0.04	0.033	0.028	0.025	0.022
1	0.0127	0.023	0.022	0.021	0.018	0.016	0.014	0.012	0.011
1 1/4	0.01588	0.012	0.012	0.011	0.01	0.009	0.008	0.007	0.006
1 1/2	0.01905	0.007	0.007	0.006	0.006	0.005	0.005	0.004	0.004

Table 1: Predicted railgun accelerations for various values of magnet thickness h and radius R (upper values in each cell) and experimentally measured values, where available (lower values).

A More Comprehensive Model

To this point, we have applied the Lorentz force law $F_m = ILB$ with a single mean value of B at all locations. In reality, the magnitude of the field is a function of position (z, r), where z is the distance along the magnet's axis and r is the radial distance from this axis. Knowledge of $B(z)$ would us to apply Newton's second law to a small current element Idz at z (Fig. 3), and then integrate to find a more realistic value for acceleration. Taking the magnets at both ends into account, Eq. (1) becomes

$$dF_m = nI[B(z) + B(L-z)]dz \ . \qquad (6)$$

We obtained an expression for $B(z)$ from kjmagnetics.com:

$$B(z) = \frac{B_r}{2}\left[\frac{h+z}{\sqrt{R^2+(h+z)^2}} - \frac{z}{\sqrt{R^2+z^2}}\right] \ , \qquad (7)$$

where B_r is the residual field strength, a constant. Because this website tabulates the values of distance z_5 at which $B(z) = 5$ G, we were able to extract values of B_r for a wide range of values of magnet radius R and thickness h via

$$B_r = \frac{10G}{\frac{h+z_5}{\sqrt{R^2+(h+z_5)^2}} - \frac{z_5}{\sqrt{R^2+z_5^2}}} \ . \qquad (8)$$

It turns out that $B_r = 1.50 \pm 0.12\ T$ for all cylindrical neodymium magnets with $^1/_{16}" \leq h \leq$ ½" and $^1/_8" \leq R \leq$ ½".

Integrating Eq. (7) from z=0 to z=L gives us the total force propelling the railgun as

$$F_m = nIB_r[R - \sqrt{R^2+h^2} - \sqrt{R^2+L^2} + \sqrt{R^2+(L+h)^2} \ . \qquad (9)$$

After reaching this point we saw that the full magnetic field in cylindrical coordinates is $B(r,z) = \hat{r}\,B_r(r,z) + \hat{z}\,B_z(r,z)$. We saw that what we really want is not $B(r,z)$ but $B_r(r,z)$ and $B_z(r,z)$. Here we write these fields as

$$B_r \simeq \frac{I\pi a^2}{c}\cos(\theta)\frac{\left(2a^2+2r^2+ar\sin(\theta)\right)}{(a^2+r^2+2ar\sin(\theta))^{5/2}} \qquad (10) \text{ and}$$

$$B_\theta \simeq -\frac{I\pi a^2}{c}\sin(\theta)\frac{(2a^2-r^2+ar\sin(\theta))}{(a^2+r^2+2ar\sin(\theta))^{5/2}} \qquad (11)$$

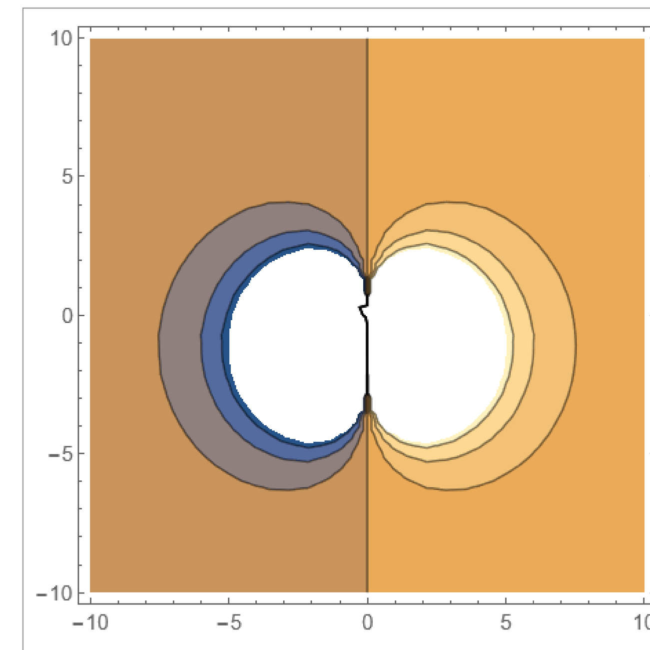


Fig. 7 Contour plot for the equation B_r. Shows current moving around a circular loop. The produced magnetic field is shown through the lines of force in the plot. Plotting B_r as a function of Cartesian coordinates, h and z.

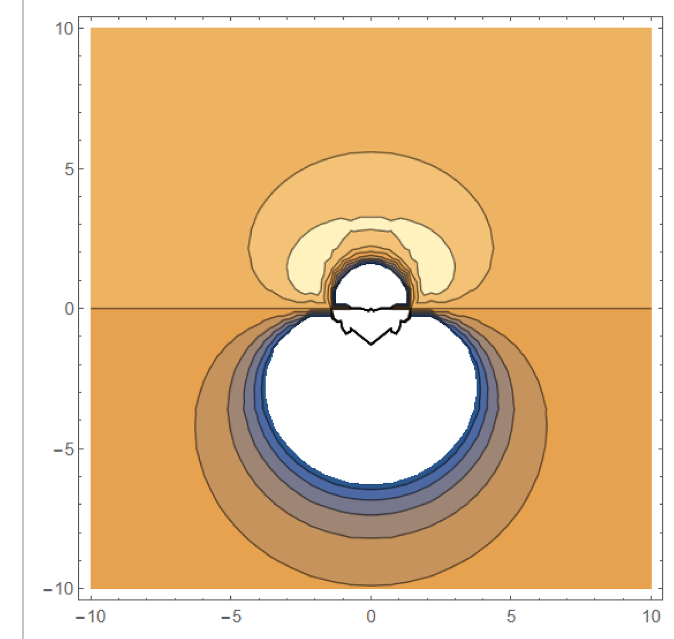


Fig. 8 Contour plot for the equation B_θ. Shows current moving around a circular loop. The produced magnetic field is shown through the lines of force in the plot. Plotting B_θ as a function of Cartesian coordinates h and z.

Using these two equations for the corresponding fields of the magnets we were able to create contour plots to help visualize the fields.

Discussion

The green highlighted entries in Table 1 show that the largest accelerations are predicted to for magnets with $2R = h = 1/8$". We plan to check this. Predicted accelerations are still about twice those we actually measured. To improve the agreement between theory and observation still further, we will need to take into account that the component of $\vec{B}$ perpendicular to the current is less than $B(z)$ as given in Eq. (7). This challenge presents an opportunity for students to connect with upper-level electromagnetism. J.D. Jackson's "Classical Electrodynamics let us see how our next goal should be suing the functions B_r and B_θ to find the component of the field that is normal to the axis. This new normal would replace $B(z)$ in our equation for the total force propelling the railguns, F_m. This moves us to a new place where we can see if we can improve the agreement between theory and observation.

Acknowledgements

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