Classroom simulation of gravitational waves from orbiting binaries

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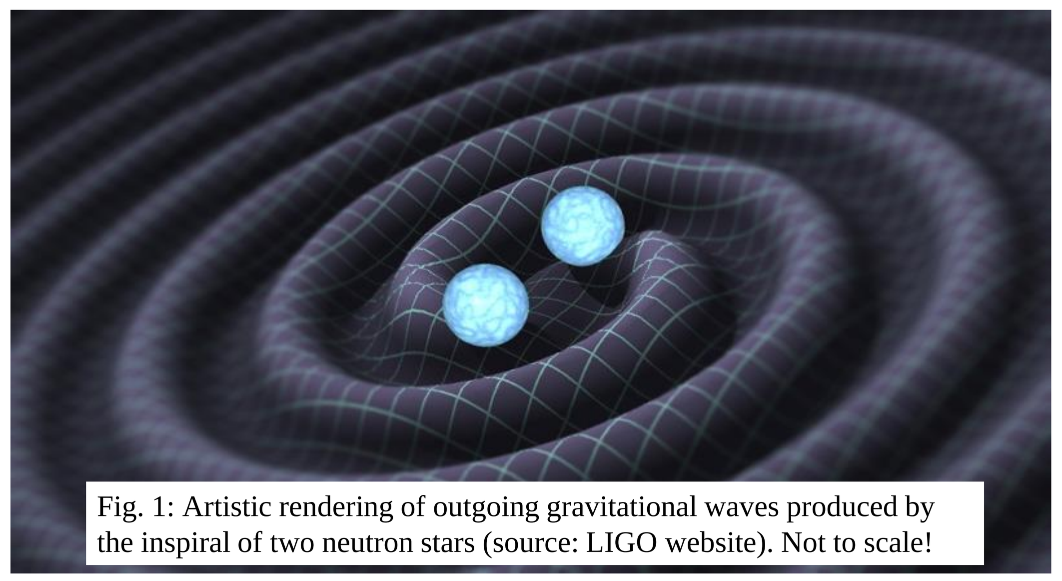

Fig. 1: Artistic rendering of outgoing gravitational waves produced by the inspiral of two neutron stars (source: LIGO website). Not to scale!

Introduction

Popular demonstrations commonly use stretched spandex fabric to illustrate the way in which curved spacetime mimics the attractive force of gravity in general relativity.[1] There are significant potential conceptual pitfalls to such an approach. In particular, it obscures the fact that most of what we ordinarily feel as gravity is due to the warping of time rather than space, a concept that is admittedly harder to demonstrate. Nevertheless, with appropriate caveats, simulations of this kind can be a memorable way to convey some of the wonder of Einstein's theory. In this spirit, we wondered whether a similar model could be used to illustrate gravitational waves from orbiting binaries, whose successful detection has been recognized with the 2017 Nobel Prize in Physics. Our simple and inexpensive demonstration reproduces the pattern of outgoing spiral ripples that has entered the public imagination through images from numerical simulations (Fig. 1).

Demonstration

"Spacetime" in our demonstration is represented by a 60" x 60" sheet of polyester Lycra 4-way spandex fabric stretched over a 60" diameter hula hoop. Hula hoops this big are hard to come by, so we made our own with a 15' length of stiff (160 psi) ¾" diameter plastic polyethylene pipe. We heated the ends of this pipe with a hair dryer until they softened enough to allow insertion of a plastic 2-way ¾" barbed connector, which then held the pipe together in an approximately circular shape when it cooled back down. We raised

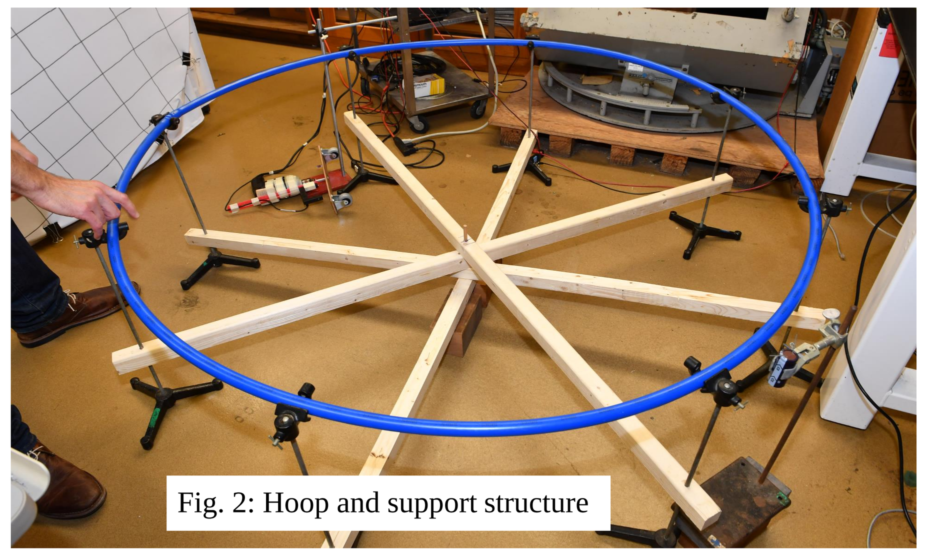

Fig. 2: Hoop and support structure

Fig. 3: Setup with "orbiting binary" including photogate velocity sensor strapped to drill (inset). DataStudio interface and strobe light are visible in the background at left and right respectively.

this hoop off the ground on a circular arrangement of 8 x 20" stands and clamps of the found in most introductory physics laboratories (Fig. 2). To ensure that the hoop remained circular we found it helpful to hold these stands apart using holes drilled in four pieces of 2" x 2" lumber arranged like the spokes of a wheel. The spandex was stretched over the hoop and held in place with binder clamps.

For our "orbiting binary" we used a pair of 1½" diameter rubber caster wheels mounted at either end of a 30 cm-long wooden crossbar (Fig. 3). We attached a ¼" hexagonal coupling nut firmly to the center of mass of this assembly so that it could be inserted into the chuck of an electric hand drill. To measure orbital speed v, we attached a Pasco photogate velocity sensor to the drill and fastened a lightweight trigger to the crossbar (Fig. 3, inset).

We expected that the amplitude of our "gravitational waves" would be largest for speeds v comparable to the characteristic wave speed (or speed of sound) c_s, which is set by the tension in the spandex. (In the same way, the amplitude of real gravitational waves reaches a maximum as the orbital speeds of the inspiraling compact objects become comparable to the speed of light.) To test this idea, we placed two small neodymium magnets on the spandex a distance Δd apart (holding them in place with paper clips on

Fig. 4: Measurement of wave speed in the spandex using neodymium magnets and induction coils (inset) connected to DataStudio interface.

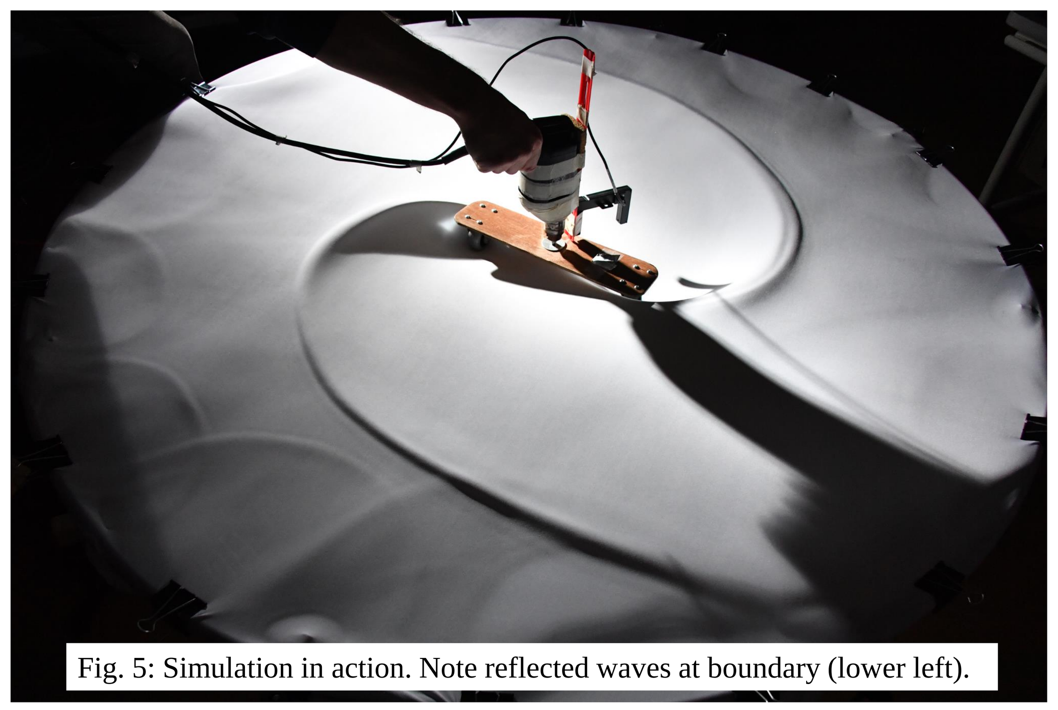

Fig. 5: Simulation in action. Note reflected waves at boundary (lower left).

the underside of the fabric) and suspended two coils immediately above them. (Fig. 4). The coils were connected to the DataStudio interface via voltage sensors. Striking the fabric next to one magnet produced a voltage spike in the coil above it (thanks to Faraday's law of induction), which then propagated to the other coil after time Δt. Trial and error showed that best results were obtained for a relatively slack fabric with $c_s = \Delta d/\Delta t \approx 3$ m/s.

To bring out the wave pattern, we used a strobe light. This also allowed us to match the orbital speed with the wave speed. For example, to achieve $v = c_s = 3$ m/s with orbital radius $R = 15$ cm, we set the strobe frequency to $\omega = v/2\pi R = 180$ rpm and gradually increased the drill speed until the crossbar appeared "frozen" (Fig. 5).

Theory

In our demonstration, it is possible for the "orbiting bodies" to move more quickly than the waves they produce. The same thing is not true in the real world, where gravitational waves travel at the speed of light c, which sets a strict upper limit on the speed of the massive bodies. This is a useful reminder that the demonstration is in no sense a relativistic one. But it also provides an opportunity to make a connection with the physics of real gravitational waves, whose amplitude falls off in 3D space according to[2]

$$h \sim \frac{GMR^2 f^2}{rc^4} \qquad (1)$$

where r = distance, R = radius and $f = \omega/2\pi$ is frequency.

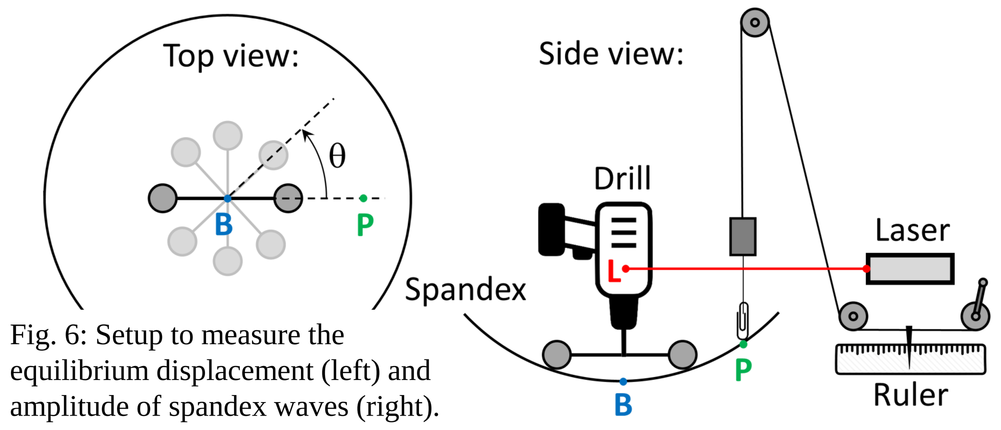

Fig. 6: Setup to measure the equilibrium displacement (left) and amplitude of spandex waves (right).

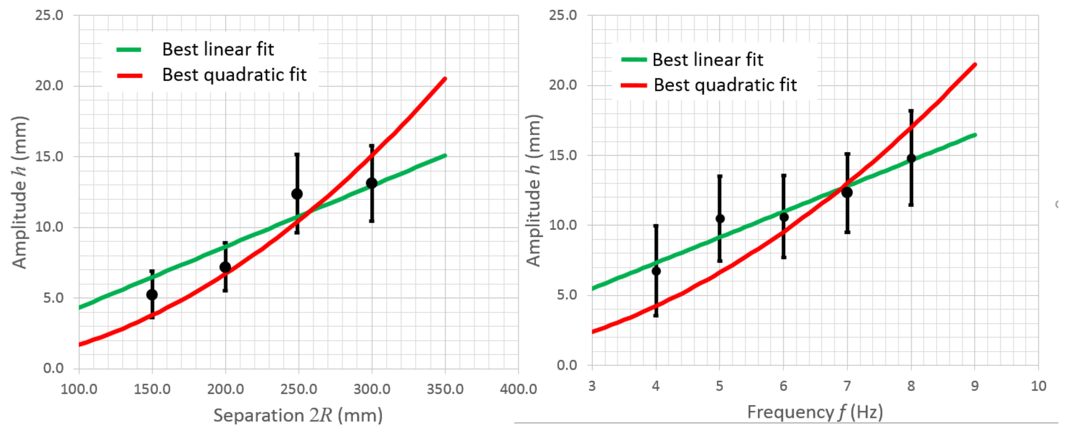

Fig. 7: Measured dependence of wave amplitude h on R (left) and f (right)

Experiment

To test whether a similar relation holds for our 2D spandex waves, we constructed an amplitude measurement device (Fig. 6). A heavy mass was suspended from a pulley in the ceiling. A light paperclip hanging from the mass made contact with the spandex (P), its position y measured to mm precision or better by a ruler. The position of the drill was kept consistent horizontally by centering it over a mark on the spandex (B) and vertically by means of a laser beam (L). Ten runs were made for each datapoint, with uncertainty given by standard deviation. Prior to each set of runs, we first measured the equilibrium displacement (no rotation) by holding the drill and crossbar at various orientations (Fig. 6, left) and averaging to find y_0. Amplitude was then given by $h = y - y_0$. Results are plotted in Fig. 7.

It is clear from Fig. 7 that amplitude in our 2D spandex fabric depends *linearly*, not quadratically on both R and f. This is interesting, because it suggests that amplitude in 2D is given by the square root of Eq. (1) for 3D waves; i.e., $h_{2D} \propto Rf/\sqrt{r}$. This makes physical sense, since the energy carried by circular 2D waves must fall off as $1/r$, not $1/r^2$ as for spherical 3D waves. And wave energy is always proportional to amplitude squared.[2] Thus, in 2D we expect that amplitude should drop as the square root of distance. We intend to test this in further experiments.

A classroom demonstration like this can only hope, at best, to simulate some aspects of gravitational waves emitted during the inspiral phase of gravitational wave production, not the more spectacular merger and ringdown phases. To understand those, students will need to learn about general relativity in its full glory.

Acknowledgments

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References

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2. L.M. Burko, "Gravitational Wave Detection in the Introductory Lab," *The Physics Teacher* 55 (2017) 288