EXOTIC SPACETIME TOPOLOGY

As an Alternative to Dark Matter and Energy

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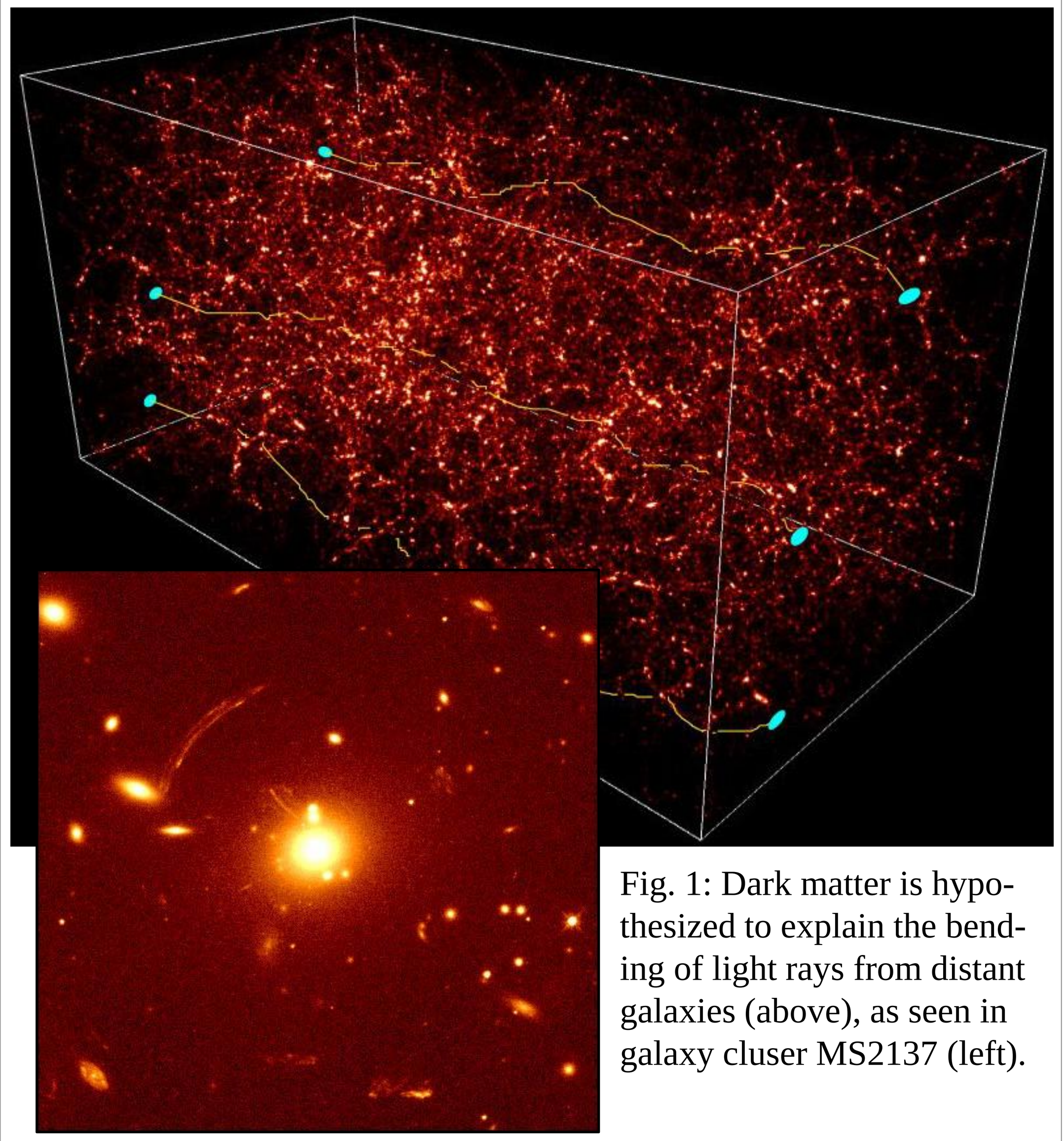

Fig. 1: Dark matter is hypothesized to explain the bending of light rays from distant galaxies (above), as seen in galaxy cluser MS2137 (left).

Dark Matter and Energy — the New Epicycles of Cosmology?

Dark matter and energy are two forms of matter-energy that are hypothesized to make up 95% of the energy content of the Universe [1]. Dark matter is gravitationally attractive (like ordinary matter) while dark energy is gravitationally repulsive and acts only on cosmological scales. Neither can consist of anything within the existing Standard Model of particle physics, and neither has been directly detected in any experiment.

The evidence for dark matter appears on many scales: individual galaxies (rotation curves), galaxy clusters (galaxy peculiar velocities, X-ray emission from hot intracluster gas, and lensing as in Fig. 1 above), and cosmology (large-scale structure formation). In each case the behavior of *visible* matter seems to require the existence of a large amount of additional matter that is *invisible*.

The evidence for dark energy is likewise indirect: its gravitationally repulsive character stretches space in a way that can explain the observed relationship between supernova magnitude and redshift. Also its energy density, when added to that of matter (both ordinary and dark) explains why the total energy density of the Universe is exactly equal to the critical density, as implied by observations of anisotropies in the cosmic microwave background.

But the fact that two invisible entities are needed to reconcile modern cosmology with observation strikes some as reminiscent of the epicycles of Ptolemaic astronomy. Einstein's General Relativity teaches us that "gravity = curved spacetime." Could it be that what we have taken as the gravitational effects of new forms of matter-energy is really just a manifestation of *exotic spacetime structure* --- i.e., topologically more complex than $\mathbb{R}^4$?

Exotic Spacetime Topology

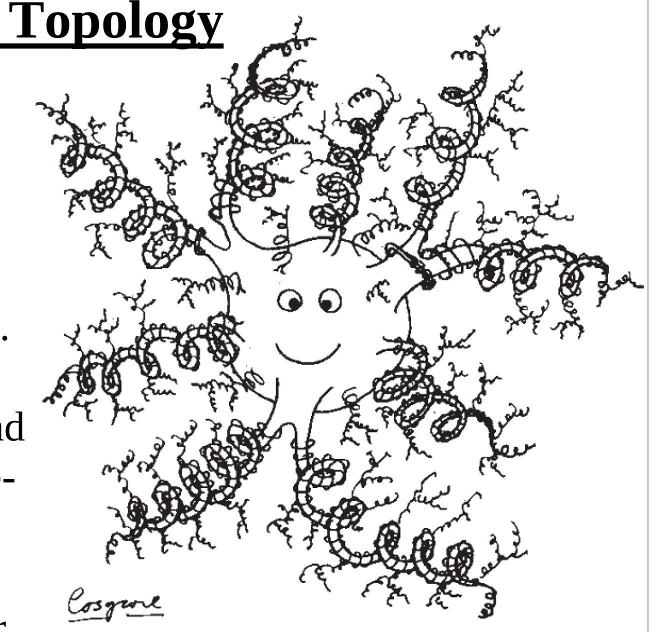

Fig. 2: An artist's interpretation of exotic four-dimensional space [2]. The center is the Euclidean representation of four-space, while the tendrils represent its exotic nature as it approaches infinity.

Topology studies the properties of space and geometry that are preserved under continuous deformations (eg. stretching or crumpling). Two such deformations are homeomorphisms and diffeomorphisms. Homeomorphisms are invertible transformations that do not involve cutting or gluing. Diffeomorphisms are differentiable homeomorphisms, where we can perform calculus.

The property of interest here is "exotic smoothness." Manifolds are topological spaces in which each point's surrounding points, called a neighborhood, locally resemble a piece of Euclidean space. Exotic manifolds are manifolds that are homeomorphic, but not diffeomorphic, to *n*-dimensional Euclidean space $\mathbb{R}^n$. Exotically smooth manifolds, in other words, are those in which we can do calculus ("smooth"), but which do not map smoothly to ordinary space ("exotic"). Such a space is depicted schematically in Fig. 2.

The first exotic space discovered was actually a seven-dimensional sphere. Discovered in 1956 by John Milnor, this object behaved like a sphere topologically, but not differentially. The first exotic versions of ordinary four-dimensional space $\mathbb{R}^4$ were discovered by Michael Freedman in 1982 and Simon Donaldson in 1983 [3]. Their work was extended by Robert Gompf in 1985 and Clifford Taubes in 1987, which showed that there are, in fact, an *uncountable infinity* of exotic versions of four-dimensional Euclidean space [4]. The points making up these manifolds can be globally topologically identified with sets of four numbers (*t*,*x*,*y*,*z*) in the usual way, and these coordinates may vary smoothly over some neighborhood — but they cannot be globally continued as smooth functions. Most remarkably of all, these exotic versions of ordinary space *exist only in the case of four dimensions*.

These results may have earthshaking physical significance, yet they have hardly been noticed by physicists so far. Is it a coincidence that real spacetime is four-dimensional? A basic requirement for any practical field theory is that the underlying space be differentiable ("smooth"). But need it be Euclidean? Einstein's theory of general relativity teaches us that what we feel as the "force of gravity" is actually just a manifestation of curved spacetime. It also seems to require vast amounts of dark matter and energy to curve that spacetime. Could it be, instead, that spacetime is simply exotic? This idea is known as the "Brans conjecture" [5].

The challenge is to extract, from the uncountable infinity of possible exotic $\mathbb{R}^4$s, a space whose properties can be described by an actual spacetime metric.

Case Study: Gravitational Lensing from Exotic Smooth Structure

In three-dimensional Euclidean space, mathematicians have been able to break down complicated manifolds into simpler, Euclidean pieces (called "handles") using something called "handlebody decomposition." Physicist Christopher Duston has used an analogous mechanism (based on work by Taubes [4]) to obtain metrics on an exotic four-manifold [7]. When this four manifold is decomposed into blocks using "Casson handling," we can give each block its own metric. Given a metric $g^{(n)}$ on each block W_n, the metric on the whole manifold can be determined using Fourier-Laplace ("Z-transform") techniques, $g_z = \sum_{-\infty}^{\infty} z^{-n} g^{(n)}$, and takes the form of what is known as an "end-periodic manifold" (Fig. 3). This mechanism gives us a way to describe curved space (and therefore gravity) in the alternate mathematical reality of exotic smooth topology.

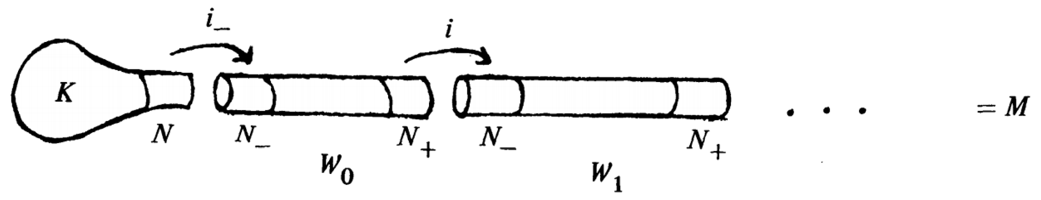

Fig. 3: An end-periodic exotic four-manifold decomposed using Casson handles, as pioneered by Taubes [4] and applied by Duston [7].

Duston has applied his method to two different models, one astrophysical (an "exotic black hole") and the other cosmological in nature ("Exotic Friedmann-Robertson-Walker" or FRW). In the former case, one obtains a version a version of the Kruskal metric of general relativity:

$$ds^2 = 2M^3 \left(\frac{(-1)^{1-n}}{(1-n)!(2M)^{1-n}} \right) (du^2 - dv^2) + r^2 d\Omega^2 \quad . \qquad (1)$$

The cosmological model yields an exotic version of the standard flat FRW metric:

$$ds^2 = -dt^2 + \tfrac{1}{2} a^2(t)(2k-1)dr^2 + r^2 a^2(t) d\Omega^2 \, . \quad (2)$$

Here $a(t)$ is the usual cosmological scale factor, but the parameter k is not the curvature parameter (this model is flat). By assuming a simple dust-like perfect fluid form for the energy-momentum tensor, Duston is able to obtain an analytic form for the scale factor,

$$a^2 = \frac{1}{8\pi G\rho} \left(1 - \frac{1}{2k-1} \right) \quad . \qquad (3)$$

Using this, he has derived a formula for gravitational light deflection in this manifold:

$$\theta = \int \frac{dr}{r^2 \sqrt{B^2 - \frac{A}{r^2}}} \quad , \qquad (4)$$

where *A* and *B* are presumably free parameters of the theory. Eq. (4) reduces to the standard light-deflection formula of general relativity for distances *r* much greater than the Schwarzschild radius. Nevertheless, this result what may be the *first empirically testable prediction of a topologically exotic alternative to Euclidean space*.

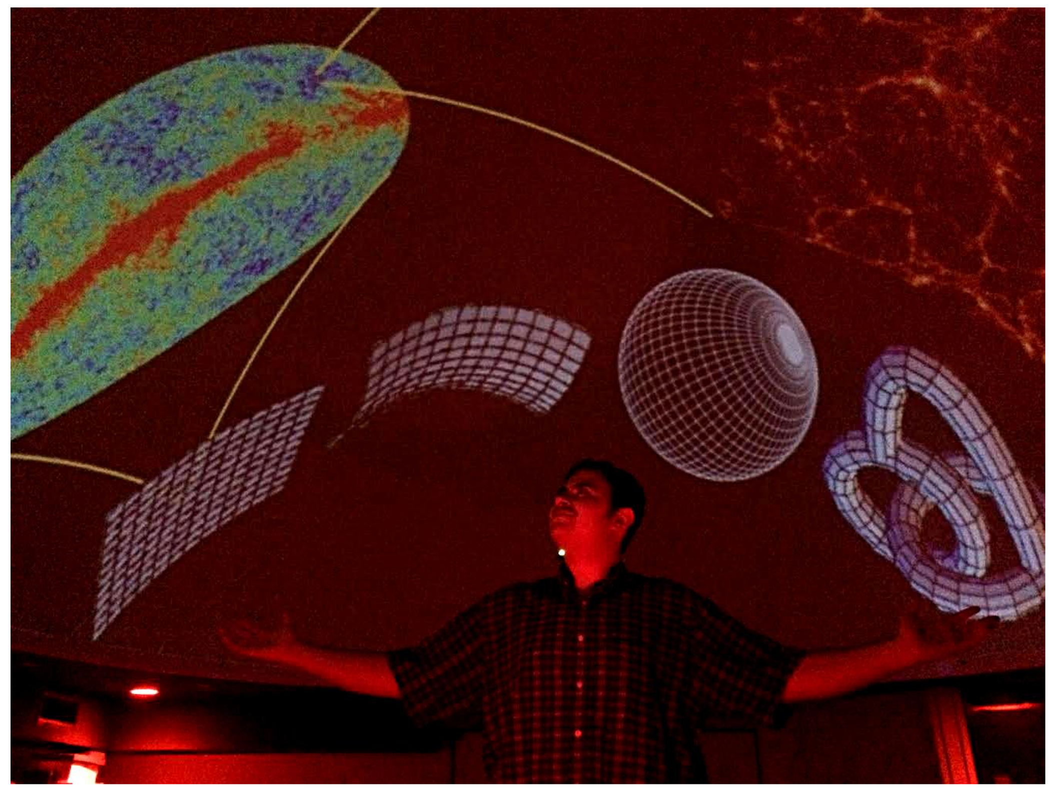

Fig. 4: G. Kuri in Towson University's planetarium, suggesting a connection between spacetime topology and cosmic background radiation.

Discussion

Many challenges remain in translating the ideas explored here into a practical alternative to dark matter or dark energy. Our next step will be to examine the implications of the metric (1) and compare them with actual data on gravitational lensing. Next, if successful, we will consider whether a model like this might be helpful in resolving some of the problems with the dark-matter hypothesis, such as density cusps and missing satellites [7].

Exotic manifolds may someday also find physical application to such fields as quantum mechanics and particle physics. Jerzy Król has suggested that the exoticness of $\mathbb{R}^4$ may provide a mathematical basis for decoherence, the mysterious process by which quantum processes on microscopic scales become classical on macroscopic ones [8]. Asselmeyer-Maluga, Król and Brans have argued that exotic smoothness might explain dark energy and inflation, and even provide a purely gravitational model for fermions [9].

Acknowledgments

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