



The Equivalence Principle

Today, two fundamental theories underlie all known physics: General Relativity (GR, governing gravity) and the Standard Model of Particle Physics (SM, governing everything else). Unfortunately, these two theories are incompatible. Unification of GR with the SM could lead to tremendous progress in science, as has happened on multiple occasions in the past.

The founding principle of GR is the Equivalence Principle (EP), which states that gravitation is locally indistinguishable from acceleration. The EP implies that all objects fall with the same acceleration in the same gravitational field, a hypothesis that was famously tested by Galileo at the Leaning Tower of Pisa (Fig. 1). But nearly all proposals to unify GR with the SM predict that objects with different compositions will, in fact, fall with slightly *different* accelerations in the same gravitational field [1]. This occurs because these theories generically predict the existence of new fields (such as dilaton and moduli fields in string theory) that interact differently with different kinds of matter in the standard model. Finding a violation of the EP, or proving that there is none, may thus be the best hope to discover the ultimate "Theory of Everything," if it exists.

One way to test the EP is to repeat Galileo's experiment inside an orbiting spacecraft, where free-falling test bodies accelerate continuously toward the Earth. ESA's *MICROSCOPE* satellite is currently doing this with test masses made of titanium and rhodium-platinum alloy [2]. Another method uses torsion balances to compare horizontal accelerations toward the Earth or Sun. The most sensitive such test so far limits any difference in relative acceleration between test masses made of aluminum, beryllium and titanium to $|\Delta a/a| < (2-9) \times 10^{-13}$ [3].

A third, "celestial method" takes advantage of the fact that Solar-System bodies are falling toward each other all the time. (What we call an "orbit" is just one body falling toward another as it moves past.) Laser ranging between the Earth and Moon currently limits any difference in the accelerations of these bodies toward the Sun to $|\Delta a/a| < (-0.8 \pm 1.3) \times 10^{-13}$ [4].

However, one might not expect strong EP violation here, because the Earth and Moon are similar in composition. The celestial method allows us to compare test bodies over a far wider range of compositions than any other test, albeit with lower precision [5]. This is important because we do not know exactly how the new, EPviolating fields predicted by unified theories will interact with ordinary matter. However, theoretical studies based on string theory suggest that three quantities may be determinative: a test body's *average baryon number* N + Z, *neutron excess* N-Z, and electrostatic nuclear binding energy $Z(Z-1)/(N+Z)^{1/3}$, where Z, N refer respectively to atomic number and neutron number [6].

Solar-System Bodies as Tests of New Physics and the special case of the all-metal asteroid 16 Psyche



Fig. 2: Selected solar-system bodies plotted in the theoretical EP-violating phase space defined by three kinds of generalized "elementary charge": average baryon number, neutron excess and nuclear binding energy. To maximize the chance of seeing an EP violation, one should compare the accelerations of bodies spanning the largest possible region of phase space. Baryon number $x \equiv 10 \left(\frac{N+Z}{M} \right)$

The Case of 16 Psyche

We have, for the first time, calculated average values of these quantities for *solar*system bodies rather than individual elements or compounds (Fig. 2, where A refers to atomic weight, and the scaling factors are for convenience. We used composition models discussed in Refs. [5,7] and [8].)

The asteroid 16 Psyche is particularly interesting, as its albedo and surface spectra suggest that it may be the only large body in the solar system to consist largely of metal. Unfortunately, the observations are uncertain at present, with a recent review concluding that its density may be anywhere between 3200 to 7600 kg/m³, with a preferred value of 4500 kg/m³ [8]. We were able to accommodate these values using a simple two-component model with an iron-nickel core (in the ratio Fe:Ni = 85:5, as for Mercury) and a silicate (SiO_2) mantle with a depth equal to 66% (Model I), 33% (Model II) and 3% (Model III) of the mean radius.

If Psyche is indeed well described by Model III, then its position at the extreme corner of this plot makes it a promising EP test body indeed. Intriguingly, recent determinations of Psyche's density (based on encounters with nearby asteroids) vary wildly, from as low as $1800 \pm 600 \text{ kg/m}^3$ [7] to as high as 7000 ± 600 kg/m³ [8]. Could this be evidence of new physics?

Modified Kepler's Third Law

To find out, we adopt a simple model for EP violation in which the gravitational mass m_a that enters Newton's law of gravity $F_a = GMm_a/r^2$ is not necessarily the same as the inertial mass m_i that enters the second law of motion $F_i = m_i a$. Equating $F_a = F_i$, we then find that acceleration is given by

$$a = \frac{GM}{r^2} \left(\frac{m_g}{m_i}\right) = \frac{GM}{r^2} (1 + \Delta), \qquad (1)$$

where $\Delta \equiv 1 - m_a/m_i$ is an EP-violating parameter

that can in principle take different values for each element in the periodic table. We then incorporate this idea into Kepler's third law of planetary motion, which states that $G(M_1 + M_2) = \omega^2 a^3$ where semi-major axis $a = R_1 + R_2$ and angular frequency or "mean motion" $\omega \equiv 2\pi/T$ (Fig. 3). The result is (2)

 $G(m_1 + m_2 + m_1\Delta_2 + m_2\Delta_1) = \omega^2 a^3$,

where Δ_1 and Δ_2 refer to the larger (central) and smaller bodies respectively. To make contact with observation we rewrite this in terms of the Gaussian constant $k \equiv Gm_{\odot}/A^3$ where A is the mean Earth-Sun distance (i.e., one AU), giving

$$\left(\frac{m_{\odot}}{m_{1}}\right)\left(\frac{\omega}{k}\right)^{2}\left(\frac{a}{A}\right)^{3} - \left(1 + \frac{1}{m_{1}/m_{2}}\right) = \frac{1}{m_{1}}$$

Kepler's third law *without* EP violation says that





Fig. 3: Key orbital parameters in the gravitational two-body problem, whose solution is expressed by Kepler's Third Law of planetary motion.

 $\frac{\Delta_1}{n_1/m_2} + \Delta_2 \quad .$ (3)

the left-hand side vanishes. Any EP violation that does occur is described by the (small) terms on the right.



(root-mean-square) sum of the *uncertainties* on the left-hand side:

$$|\Delta_2| \le \left[\left[\frac{\delta(m_{\odot}/m_1)}{m_{\odot}/m_1} \right]^2 + \left(2\frac{\delta\omega}{\omega} \right)^2 + \left(3\frac{\delta a}{a} \right)^2 + \left(3\frac{\delta A}{A} \right)^2 + \left[\frac{\delta(m_1/m_2)}{(m_1/m_2)^2} \right]^2 \right]^{1/2}.$$
 (4)

We focus on Psyche's orbit around the Sun, so $m_2 = m_{16P}$ and $m_1 = m_{\odot}$ (thus the first term vanishes by definition). Recent asteroid ephemeris observations give $\omega \pm \delta \omega = 2.6 \times 10^7 \pm 0.076$ arcseconds/century, $a \pm \delta a = 2.9 \pm 6 \times 10^{-9}$ AU, and $A \pm \delta A = 1.5 \times 10^{11} \pm 3$ m [12]. From Ref. [11], $\mu^{-1} = 3.38 \times 10^{-11}$ and $\delta(\mu^{-1}) = 0.28 \times 10^{-11}$ where $\mu \equiv m_{\odot}/m_{16P}$, so that $\mu \pm \delta \mu = 2.96 \times 10^{10} \pm 10^{10}$ 2.5×10^9 . Putting these values into Eq. (4), we find $|\Delta_{16P}| \le 9 \times 10^{-9}$

dominated by the uncertainties in ω and a. This limit is not as sensitive as those obtained from torsion balances or lunar laser ranging, but it is significant because it broadens the range of test materials considered in EP tests to date. (Similar, though weaker conclusions would apply to Mercury, which has a large metallic core.) This sensitivity may improve following NASA's planned visit to Psyche, currently set to launch in 2022 (Fig. 4). But ultimately, the best way to derive limits on EP violation for solar-system bodies is to include additional parameters in the fitting procedure that generates the solar-system ephemeris itself.

Acknowledgments

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References

- [1] J. Overduin et al., "STEP and fundamental physics," *Classical and Quantum Gravity* 29 (2012) 184012
- Quantum Gravity 29 (2012) 184004 and Quantum Gravity 31 (2014) 015001
- Rencontre de Moriond (Singapore: Éditions Frontières, 1994), p. 433
- [7] R. Helled et al., "Interior models of Uranus and Neptune," Astrophysical Journal 726 (2011) 15
- thermal infrared," *Icarus* 226 (2013) 419 [9] M.K. Shepard et al., "Radar observations and shape model of asteroid 16 Psyche," Icarus 281 (2017) 388
- [11] M. Kuzmanoski and A. Kovačević, "Motion of the asteroid (13206) 1997GC22 and the mass of (16) Psyche," Astronomy and Astrophysics 395 (2002) L17
- [12] J. Hilton, "US Naval Observatory ephemerides of the largest asteroids," Astronomical Journal 117 (1999) 1077





We wish to see how large Δ_2 might be for a small, metallic body like Psyche. We neglect $\Delta_1/(m_1/m_2)$ by comparison because $m_1 \gg m_2$, and because the value of Δ_1 for non-metallic bodies is tightly constrained by the precision of NASA's current solar-system ephemeris [5]. Then, since the left-hand side of Eq. (3) is zero (Kepler's third law), our experimental upper limit on $|\Delta_2|$ is simply the statistical

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[2] M. Rodrigues et al., "MICROSCOPE: a micro-satellite for a major corner stone in fundamental physics," Small Satellites, Systems and Services Symposium 2016; https://hal.archives-ouvertes.fr/hal-01441728/document (accessed 10/23/2017) [3] T.A. Wagner et al., "Torsion-based tests of the weak equivalence principle, *Classical and Quantum Gravity* 29 (2012) 184002 [4] J.G. Williams, S.G. Turyshev and D.H. Boggs, "Lunar laser ranging tests of the equivalence principle," Classical and

[5] J. Overduin, J. Mitcham and Z. Warecki, "Expanded solar-system limits on violations of the equivalence principle, Classical

[6] T. Damour and J.-P. Blaser, Optimizing the choice of materials in equivalence principle experiments," in proc. XXIXth

[8] A. Matter et al., "Evidence of a metal-rich surface for the asteroid (16) Psyche from interferometric observations in the

[10] B. Viateau, "Mass and density of asteroids (16) Psyche and (121) Hermione," Astronomy and Astrophysics 354 (2000) 725